# CS375 Midterm Exam Solution Set (100 points)

Closed book & Closed Notes March 14, 2025

Name _____

1. (4 points)

(i) For the following regular expression find a language (i.e., a set of strings) over A = {a, b, c} that can be represented/described by this expression. Put your answer in the following blank. (2 points)

$a^*bc + ab^*$

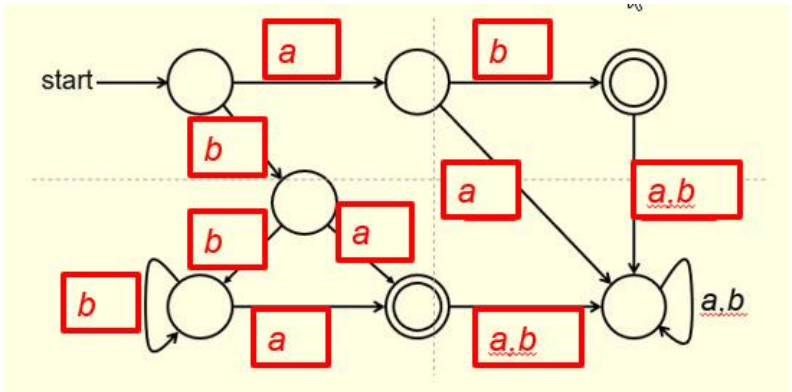
$\{a^m bc \mid m \in N\} \cup \{ab^n \mid n \in N\}$

(ii) If a regular expression for the language over the alphabet {a, b} with no string containing the substring bb is (a+ba)*(Λ+b), then what is the regular expression for the language over the same alphabet with no string containing the substring bbb? Put your answer in the following blank. (2 points)

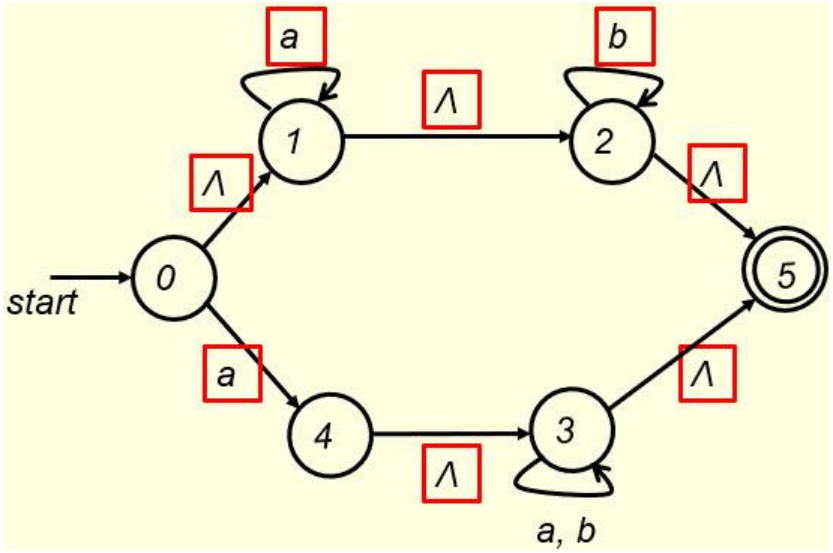
(a + ba + bba)*(Λ + b + bb).

2. (9 points)

(i) Fill out the blanks in the following figure to make it a DFA over A={a, b} that recognizes the expression ab + bb*a. (5 points)

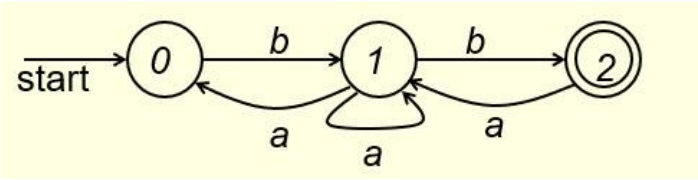



(ii) Fill out the blanks in the following figure to make it an NFA over A = {a, b} for the expression b*+a*b*+a(a+b)* (4 points)

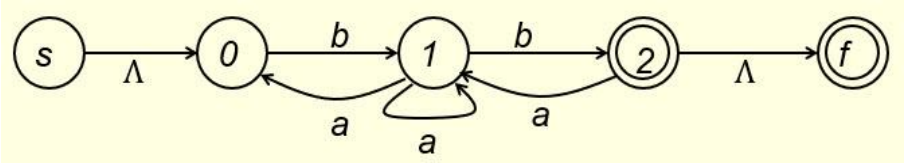



3. (10 points)

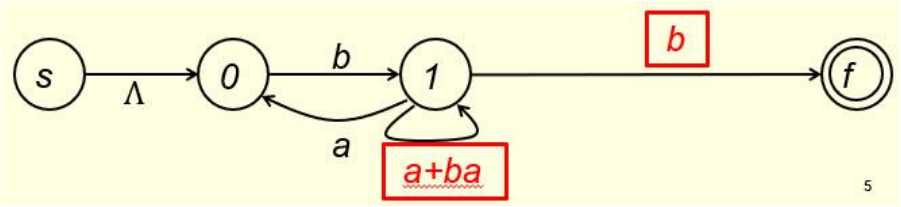
In the process of transforming the following NFA to a regular expression,




we first connect a new start state s to the start state of the given NFA and connect each final state of the given NFA to a new final state f as shown below.

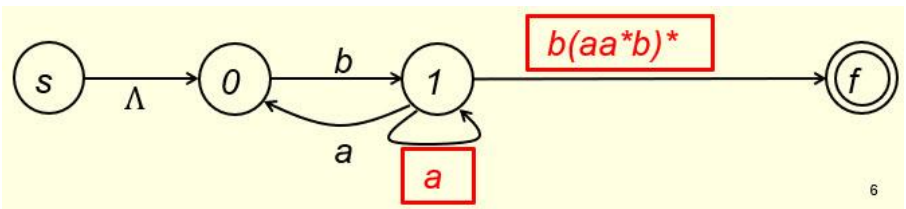



If we eliminate state 2 first, the modified NFA becomes of the following form. Fill out the blanks in the following figure. (4 points)



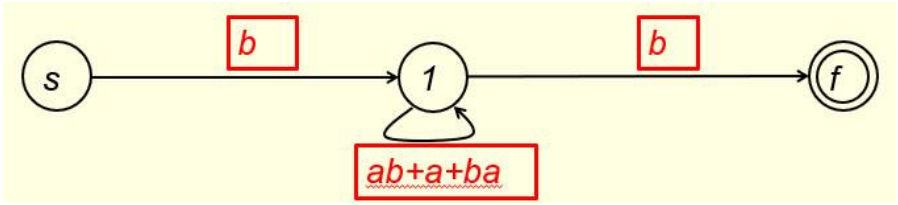

……(*)

or



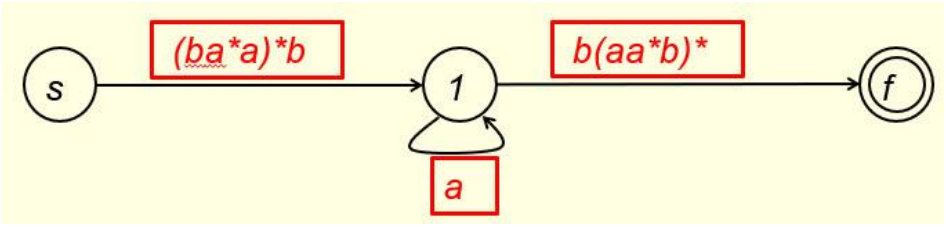

……(**)

If we eliminate state 0 in (*) then, we get the following NFA. Fill out the blanks below. Only the new blanks will be graded. (4 points)



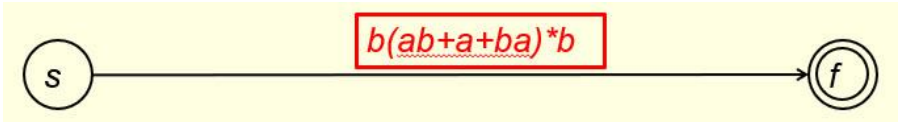
……($)

If we eliminate state 0 in (**) then, we get the following NFA. Fill out the blanks below. Only the new blanks will be graded. (4 points)



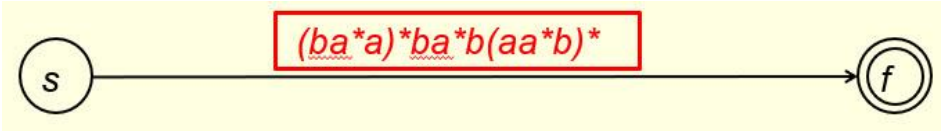
……($$)

By eliminating state 1 in ($), we get the following NFA. Fill out the blank below (2 points).



……(#)

By eliminating state 1 in ($$), we get the following NFA. Fill out the blank below (2 points).

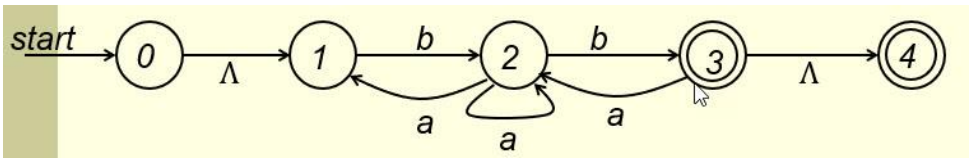


……(##)

The expressions in (#) and (##) are both the regular expression of the given NFA. You only have to do one case here, either (*), ($) and (#), or (**), ($$) and (##).

4. (18 points)

Given the following NFA over the alphabet {a, b},



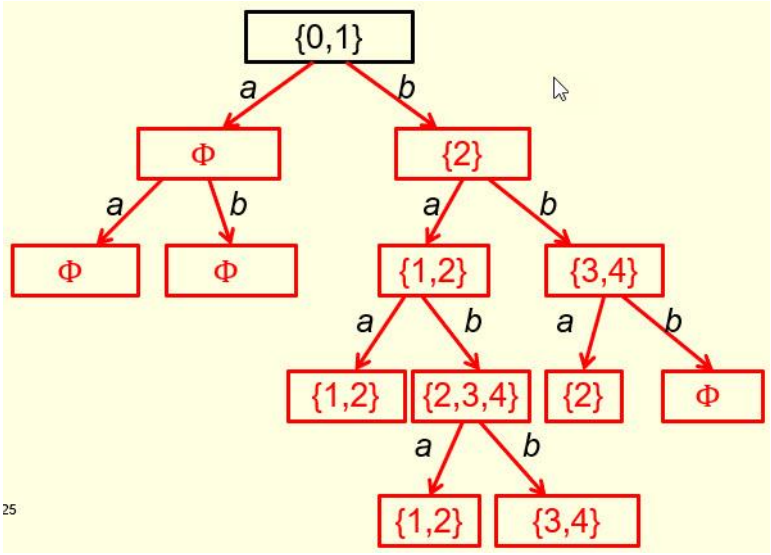

to transform it to a DFA, first construct Λ-closures of the states of the NFA. Fill out the blanks for state 0, state 1, state 4 and Φ below. (2 points)

$\Lambda(0) = \{0, 1\}$ $\Lambda(1) = \{1\}$

$\Lambda(2) = \{2\}$ $\Lambda(3) = \{3, 4\}$

$\Lambda(4) = \{4\}$ $\Lambda(\Phi) = \Phi$

Next, to get the states of the DFA, we construct a tree as follows. Fill out the blanks in the following tree. (6 points)




The distinct nodes in the above binary tree are listed below (for each level, nodes are taken from the tree from left to right). Fill out the blanks. (2 points)

{0,1} | Φ | { 2 } | {1, 2} | {3, 4} | {2,3,4}

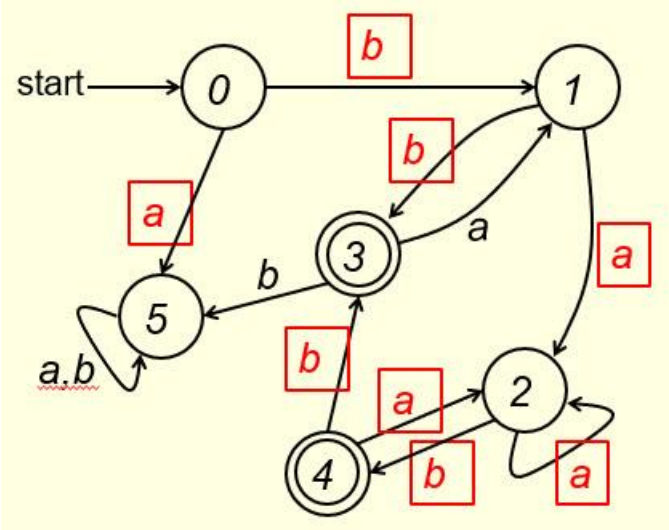
These are the states of the DFA to be constructed and we have the following transition table for the DFA. Fill out the blanks in the table. (4 points)

| | $T_D$ | a | b |
|---|---|---|---|
| S | {0,1} | Φ | {2} |
| | {2} | {1,2} | {3,4} |
| | {1,2} | {1,2} | {2,3,4} |
| F | {3,4} | {2} | Φ |
| F | {2,3,4} | {1,2} | {3,4} |
| | Φ | Φ | Φ |

Now replace the six states from top down with 0, 1, 2, 3, 4 and 5, respectively, the transition table is of the following form. Fill out the blanks in the table. (2 points)

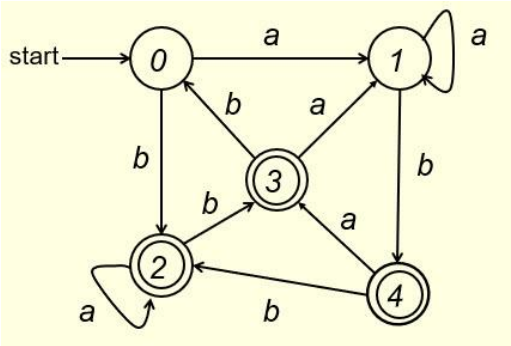
| | $T_D$ | a | b |
|---|---|---|---|
| S | 0 | 5 | 1 |
| | 1 | 2 | 3 |
| | 2 | 2 | 4 |
| F | 3 | 1 | 5 |
| F | 4 | 2 | 3 |
| | 5 | 5 | 5 |

Hence, a DFA can be constructed as follows. Fill out the blanks in the DFA. (2 points)

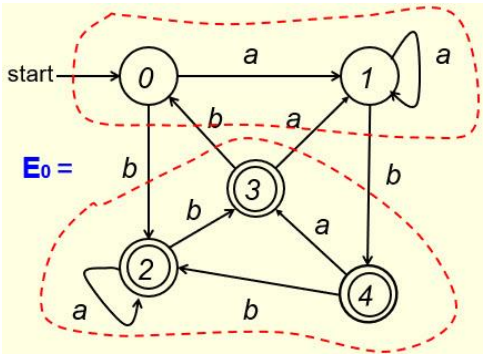


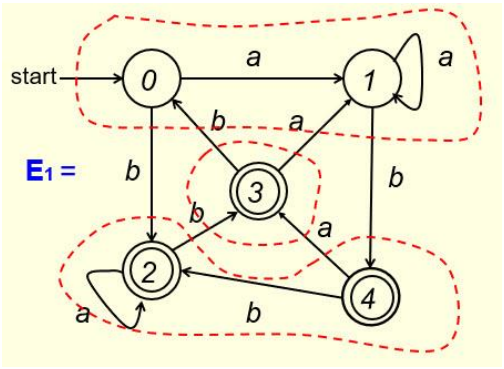

5. (4 points)

(i) Given the following DFA, to find a minimum-state DFA, (2 points)



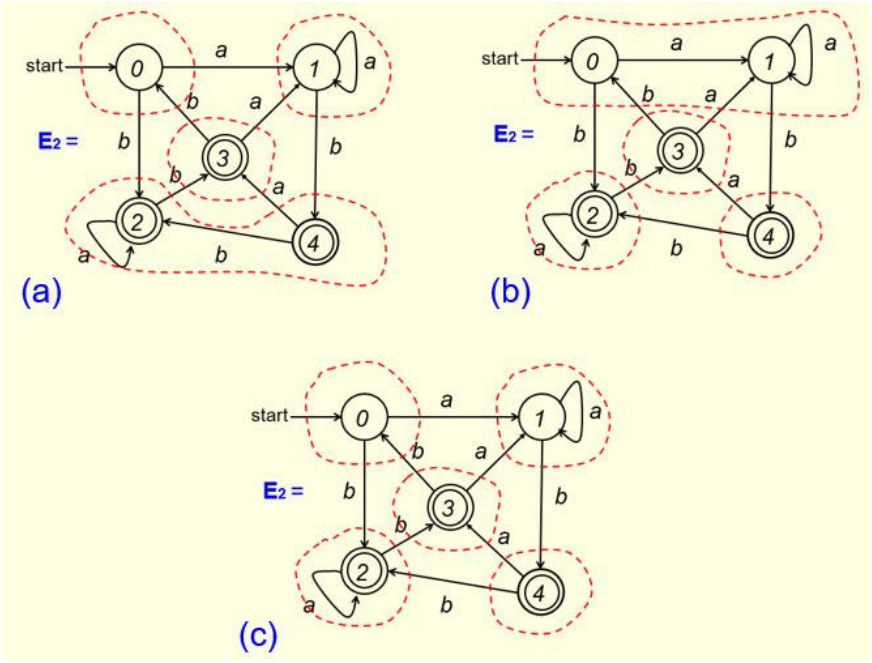

we first construct $E_0$ as follows and then $E_1$




Since $E_1 \neq E_0$, so we need to construct $E_2$. Out of the following three cases (a), (b) and (c), which one represents the correct $E_2$? Put your answer in the following blank.

The correct $E_2$ is : (b)




(ii) Let the set of states for a DFA be *S* = {0, 1, 2, 3, 4, 5}, where the start state is 0 and the final states are 1, 3 and 5. Let the equivalence relation on *S* for a minimum-state DFA be generated by the following set of equivalent pairs of states:

{ (0, 2), (1, 3) }

The states of the minimum-state DFA are: (2 points)

| {0,2} | {1, 3} | {4} | {5} |
|---|---|---|---|

(set notations)

or

| [0] | [1] | [4] | [5] |
|---|---|---|---|

(equivalence class notations)

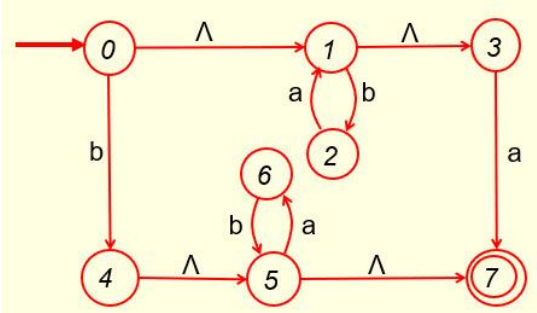
You only need to answer one of the above two cases.

6. (16 points)

Given the following regular expression over the alphabet {a, b},

(ba)*a + b(ab)*

to transform it to a regular grammar, we first transform it to an NFA as the one shown below.

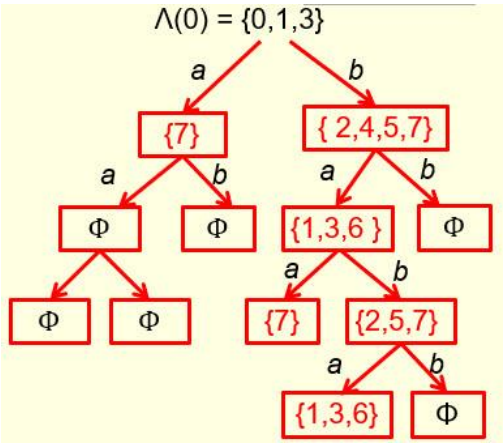



Instead of converting this NFA to a regular grammar directly, we convert it a DFA first and then convert the DFA to a regular grammar. So we construct Λ-closures of the above NFA,

Λ(0) = {0,1,3} Λ(1) = {1,3} Λ(2) = {2} Λ(3) = {3} Λ(4) = {4,5,7}
Λ(5) = {5,7} Λ(6) = {6} Λ(7) = {7} Λ(Φ) = Φ

,

and build the following tree, (3 points)




so that the following nodes of this tree can be used to build a DFA. Fill out the following blanks and the blanks in the above tree.

| {0,1,3} | {7} | {2,4,5,7} | {1,3,6} | {2,5,7} | Φ |
|---|---|---|---|---|---|

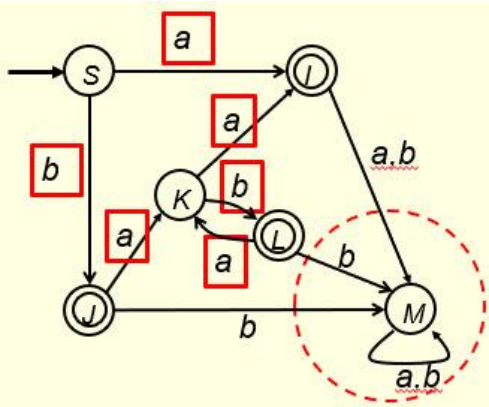
(2 points)

With the selected nodes from the above tree, we build a DFA by constructing the following transition table (the table on the left), and renaming the states as S (start state), I, J, K, L and M: (3.5 points)

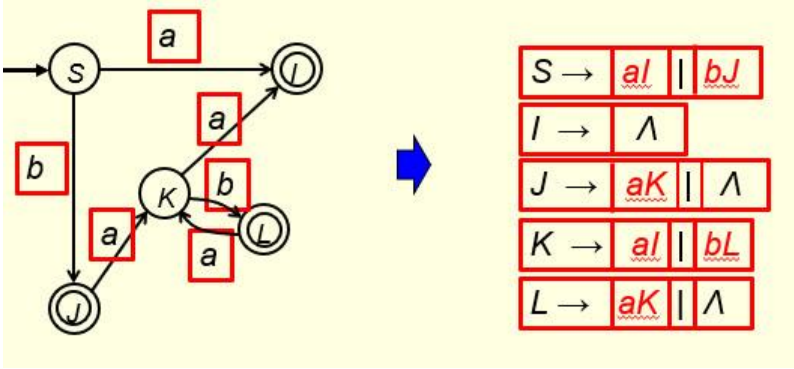
| | T | a | b |
|---|---|---|---|
| S | {0,1,3} | {7} | {2,4,5,7} |
| F | {7} | Φ | Φ |
| F | {2,4,5,7} | {1,3,6} | Φ |
| | {1,3,6} | {7} | {2,5,7} |
| F | {2,5,7} | {1,3,6} | Φ |
| | Φ | Φ | Φ |

| | T | a | b |
|---|---|---|---|
| S | S | I | J |
| F | I | M | M |
| F | J | K | M |
| | K | I | L |
| F | L | K | M |
| | M | M | M |

Then convert the table representation to a digraph representation as follows. Fill out the blanks in the following digraph. (3 points)



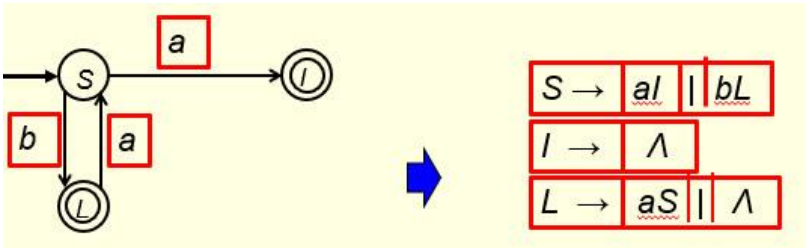

However, we don't need the portion circled in the above digraph representation of the DFA. By removing that portion, the DFA is like the one shown on the left side of the following figure.




By constructing production rules from this FA, we get the production set of the regular

grammar on the right side of the above figure. Fill out the blanks in the above figure. (3 points)

The above FA can be further simplified into a form like the one shown below (left hand side of the following figure). (1.5 points)

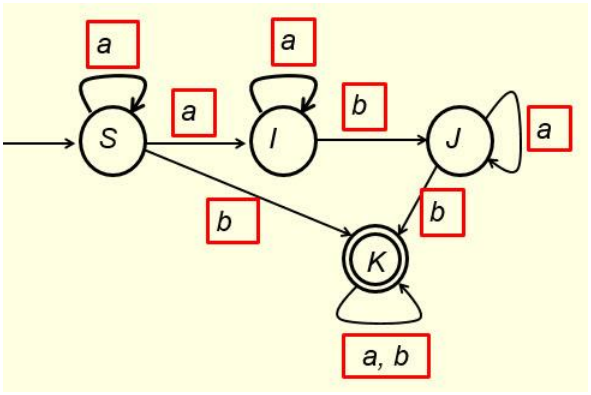



By constructing production rules from this FA, we get the production set on the right side of the above figure (1.5 points). This is the production set of the regular grammar for the regular expression (ba)*a+b(ab)* = (ba)*(a+b).

7. (4 points)
Given the following regular grammar:

S → aS | aI | bK, I → bJ | aI, J → bK | aJ, K → aK| bK | Λ

one can use the algorithm given in the notes “Regular Languages and Finite Automata- IV” to construct an NFA to recognize the language of this grammar. Fill out blanks in the following figure so that the resulting NDA would fulfill such a task, i.e., would recognize the language of the given grammar.




8. Fill out the following blanks to make it a regular grammar for the given regular expression with S being the start symbol: a*b*c* + d (5 points )

S → d | aA | bB | cC | Λ
A → aA | bB | cC | Λ
B → bB | cC | Λ
C → cC | Λ

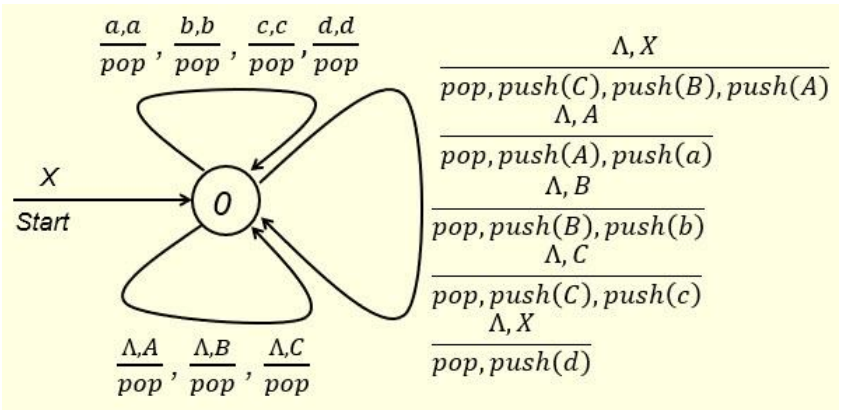
or

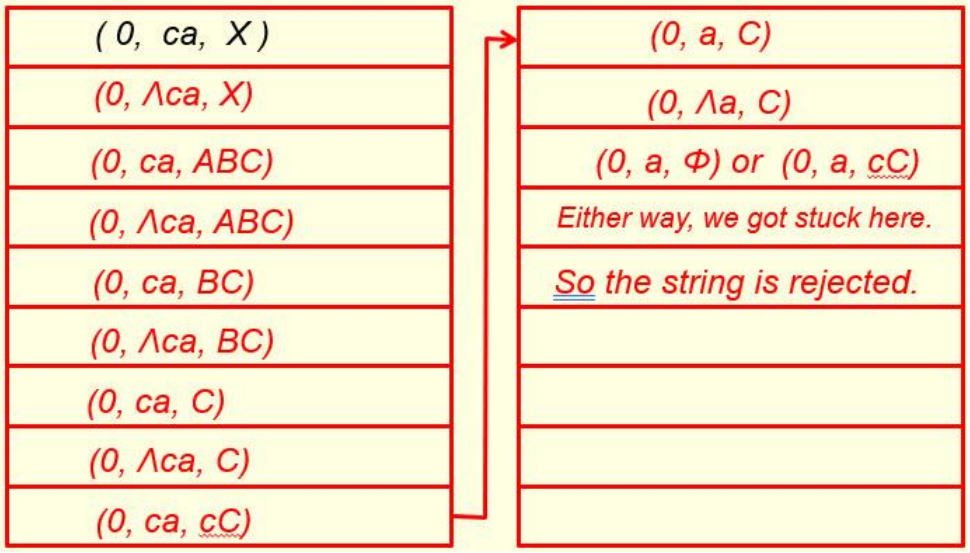
S → d | A | B | C | Λ
A → aA | B | C | Λ
B → bB | C | Λ
C → cC | Λ

9. Fill out the following blanks to make it a context-free grammar for the given language over the alphabet {a, b}: $\{ a^{2n+1}b^n \mid n >= 0\}$ (4 points)

S → aaSb | a

10. (6 points)
Given the following empty-stack PDA, check if the string *ca* would be accepted by this PDA. Show your work in the following table.






11. (8 points)

The language generated by the following C-F grammar

$$S \rightarrow ABC \mid d \qquad A \rightarrow aA \mid \Lambda \qquad B \rightarrow bB \mid \Lambda \qquad C \rightarrow cC \mid \Lambda$$

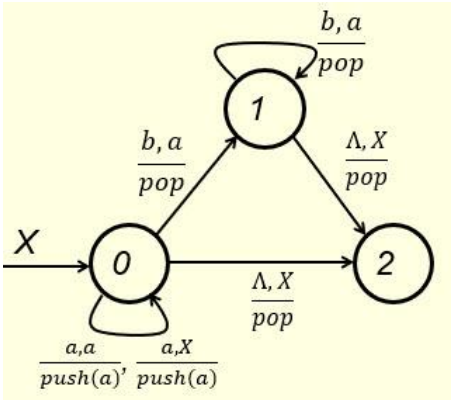
is $\{a^m b^n c^k + d \mid m, n, k \in N\}$. To convert the above C-F grammar to a one-state empty-stack PDA so that the accepted language of the PDA is the same as the language generated by this C-F grammar, we need to construct a set of specific instructions for this PDA. Four instructions have already been given below. Construct the remaining instructions for this PDA in the following blanks.

(0, a, a, pop, 0)
(0, b, b, pop, 0)
(0, c, c, pop, 0)
(0, d, d, pop, 0)

(0, Λ, S, <pop, push(C), push(B), push(A)>, 0)
(0, Λ, C, <pop, push(C), push(c)>, 0)
(0, Λ, C, pop, 0)
(0, Λ, B, <pop, push(B), push(b)>, 0)
(0, Λ, B, pop, 0)
(0, Λ, A, <pop, push(A), push(a)>, 0)
(0, Λ, A, pop, 0)
(0, Λ, S, <pop, push(d)>, 0)

12. (12 points; 8 points for the first eight blanks)

Given the following empty-stack PDA,

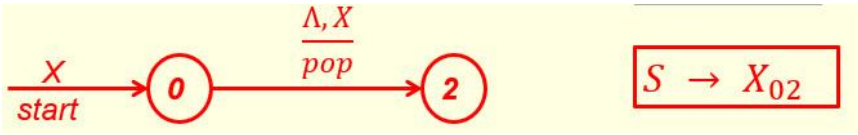



the language accepted by the empty-stack PDA is $L = \{a^n b^n \mid n \in N\}$. To convert the PDA to a C-F grammar, for each type of the PDA instructions, we need to construct the corresponding grammar productions.

In the following, I list the PDA instructions on the left and you put the corresponding grammar productions in the blanks on the right. We start with Type 4.

Type 4:

The start state and the PDA instruction Λ,X/pop provide two type 4 paths:





Type 1:

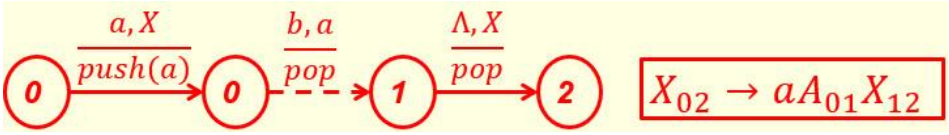
The PDA instruction Λ,X/pop by itself gives two type 1 cases:


The PDA instruction b,a/pop by itself gives two type 1 cases:

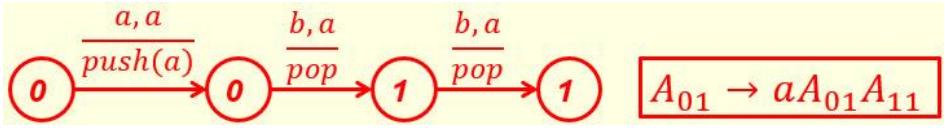


Type 3:

The PDA instruction a,X/push(a) together with the instructions b,a/pop and Λ,X/pop as shown below gives:




The PDA instruction a,a/push(a) together with the instruction b,a/pop (used twice) as shown below gives:




There are no Type 2 productions because there are no PDA instructions that perform nop stack operation.

Collecting all the productions, we get a context-free grammar whose production set is as follows:

| | |
|---|---|
| $S$ | $\rightarrow X_{02}$ |
| $X_{02}$ | $\rightarrow \Lambda$ |
| $X_{12}$ | $\rightarrow \Lambda$ |
| $A_{01}$ | $\rightarrow b$ |
| $A_{11}$ | $\rightarrow b$ |
| $X_{02}$ | $\rightarrow aA_{01}X_{12}$ |
| $A_{01}$ | $\rightarrow aA_{01}A_{11}$ |

(0 points)

After simplification, we get

| | | |
|---|---|---|
| $S \rightarrow$ | $\Lambda$ | $aA$ |
| $A \rightarrow$ | $b$ | $aAb$ |

(4 points)

and it is easy to see that this is a C-F grammar for *L*.