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# Mesh Simplification by Volume Variation with Feature Preserving

**Abstract** A new algorithm for mesh simplification with triangle constriction is presented in this paper. Constricting error defined by a combination of square volume error variation with constraint (SVEC), shape factor and normal constraint factor of triangle. Gauss curvature factor of each constricted triangle is used to distinguish strong feature triangle or non-strong feature triangle. The triangle which has minimum constricting error is constricted firstly. For non-strong feature triangle, new vertex position is determined by minimizing the constriction error of mesh model. For strong feature triangle, the vertex with maximal absolute Gauss curvature of the three triangle vertices is used as the new vertex. Experiments show the efficiency of the new algorithm.

**Keywords** Mesh simplification; Triangle constriction; QEM; SVEC; Feature factor; Gauss curvature

## 1 Introduction

Polygonal meshes are widely used in computer graphics, visualization and CAD/CAM systems. Triangular mesh and quadrangular mesh are two common mesh presentation forms, and triangular mesh is used more widely for its simple expression and steady topology. At present, by using precise laser range scanners, it is possible to obtain models more complex. However, for the complex models, the current graphics system is hard to handle them. As models get larger, they are more difficult to store, transfer, render and modify. One of the best solutions is to represent the complex model in multiple levels: multi-resolution models via mesh simplification.

Mesh simplification is to use less mesh to express the original models, and to keep a less distortion. There are many methods for mesh simplification [1-8], in which, the geometric elements deletion method is more intuitionistic and effective. The geometric elements deletion method includes vertex deletion, edge constriction and triangle constriction. In 1992, Schroedor presented a method for mesh simplification based on vertex decimation [1]. Hoppe presented an edge constriction method [2] based on energy optimization, but this method is too complicated to simplify model in an appropriate time. Garland [3] presented a new method for mesh simplification based on quadratic error metric. This method has a fast computing speed and can achieve ideal simplification result. QEM method is now a widely accepted method for mesh simplification. However, QEM method only defined the constriction error as the distance from vertex to plane, the mesh geometric properties such as triangle shape and volume variation of the model are not considered at all. Some later researches use amended QEM methods for mesh simplification [8-11].

In this paper, we present a new method for mesh simplification based on triangle constriction. The square volume error variation with constraint (SVEC) is used as an objective function for triangle constriction. SVEC combine with triangle shape factor and normal constraint factor to define the constriction error of a triangle. In simplifying process, Gauss curvature filter factor is used to judge the strong feature triangles so that the strong feature triangles are preserved after mesh being simplified. The simplified mesh by the new method has the properties that 1) the shapes of the triangles are related uniform; 2) with the constraint, when a triangle is deleted, the variation of volumes of the tetrahedrons is also related uniform.

The remainder of this paper is organized as follows: Section 2 reviews related work and presents the three basic geometric simplification processes adopted by most simplification algorithms. In Section 3, we introduce some notation to formally describe the basic simplification process we employ and present our simplification scheme. Section 4 shows some experimental results of the new algorithm, and contrasting them with results of other algorithms. In Section 5, we give the conclusion and future work.

## 2 Related work

The existing mesh simplification algorithms can be broadly classified into geometric-based and appearance-based. Most of the appearance-based methods can in fact be extended from the geometric-based methods by incorporating visual or appearance criteria. So we only focus on geometric-based simplification algorithms. The geometric-based simplification algorithms can generally classified into three different basic simplification processes:

**§ Vertex Decimation**: A vertex and its surrounding region are deleted, and the resulting hole is filled with re-triangulation (see as Fig. 1(a)). Most early simplification methods are based on this process. Schroedor [1] used the distance from a vertex to the average plane of its surrounding vertices to order the vertex decimation**.**

**§ Edge Constriction**: An edge is constricted into a new vertex. One constriction step can delete two adjacent triangles (see Fig. 1(b)). Edge constriction is the most common method and has been extensively researched. Garland presents a simplification scheme based on GEM (Quadric Error Metric), and achieves mesh simplification with edge constriction. Optimal new vertex placement is achievable under this metrics.

**§ Triangle Constriction**: A triangle is constricted to a new vertex and the adjacent triangles of this triangle are immerged (see Fig. 1(c)). This process is a little more complex than the previous two, and there are some researches on it. Hamann presents a method for estimate triangle curvature. Curvature and angles of triangle are used to compute constriction cost. Gieng defines the constriction cost as the product of triangle area and curvature. But both of these two methods have high computation cost. Zhou presents a triangle constriction method based on QEM [8], and a constriction cost transmit method is used for revising the related triangles.

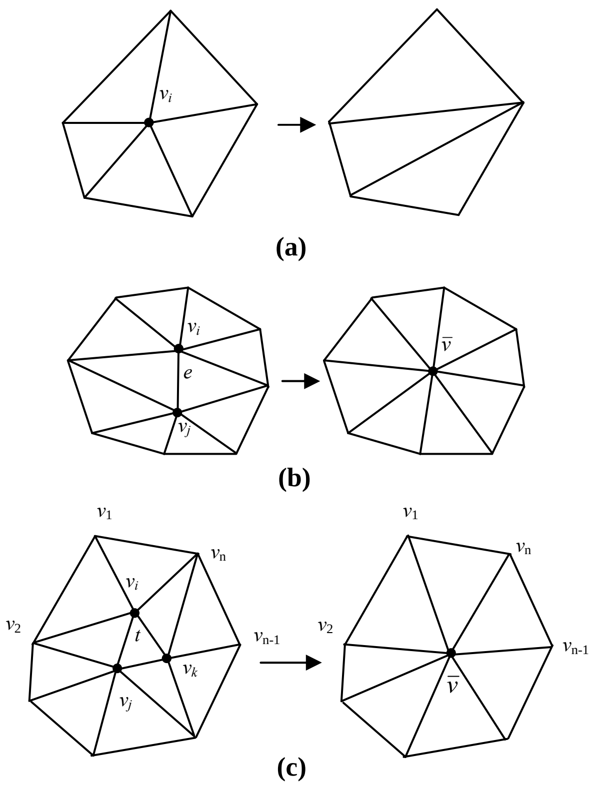



**Fig. 1 (a)** Mesh simplification based on vertex decimation; **(b)** Mesh simplification based on edge constriction; **(c)** Mesh simplification based on triangle decimation

2.1 Quadric Error Metric

QEM is first used for edge constriction mesh simplification in [3]. In order to achieve mesh simplification sequence, each vertex $v_i = [x_i, y_i, z_i, 1]^{\mathrm{T}}$ of original mesh is distributed a symmetric error matrix $\mathbf{Q}_i$. The error matrix $\overline{\mathbf{Q}}$ of each edge is the sum of error matrix of two vertices $\overline{\mathbf{Q}} = \mathbf{Q}_i + \mathbf{Q}_j$. Suppose the new vertex after constricting an edge $e$ be $\overline{v} = [\overline{x}, \overline{y}, \overline{z}, 1]$, then, the constriction error can be defined as follows:

$$\begin{aligned}\Delta(\overline{v}) = \overline{v}^{\mathrm{T}}\overline{\mathbf{Q}}\overline{v} &= \overline{q}_{11}\overline{x}^2 + 2\overline{q}_{12}\overline{xy} + 2\overline{q}_{13}\overline{xz} + 2\overline{q}_{14}\overline{x} + \overline{q}_{22}\overline{y}^2 \\ &+ 2\overline{q}_{23}\overline{yz} + 2\overline{q}_{24}\overline{y} + \overline{q}_{33}\overline{z}^2 + 2\overline{q}_{34}\overline{z} + q_{44}\end{aligned} \tag{1}$$

The position of new vertex can be determined by minimizing (1), namely,

$\partial\Delta(\overline{v})/\partial\overline{x} = \partial\Delta(\overline{v})/\partial\overline{y} = \partial\Delta(\overline{v})/\partial\overline{z} = 0$

We can reset the above equations system as:

$$\bar{v} = \begin{bmatrix} \bar{q}_{11} & \bar{q}_{12} & \bar{q}_{13} & \bar{q}_{14} \\ \bar{q}_{12} & \bar{q}_{22} & \bar{q}_{23} & \bar{q}_{24} \\ \bar{q}_{13} & \bar{q}_{23} & \bar{q}_{33} & \bar{q}_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

In QEM method, the unknowns in (1) can be obtained with the sum of the distances from vertex to its adjacent triangles. In Fig. 1 (b), we define the adjacent triangle plane of vertices $v_i, v_j$ is $ax+by+cz+d=0$, where $a^2+b^2+c^2=1$. Let $\mathbf{p}=(a,b,c,d)$, then the distance of new vertex $\bar{v}$ to the triangle plane is $\mathbf{p}^{\mathrm{T}}\bar{v}$, (1) can be re-written as:

$$\Delta(\bar{v}) = \sum(\bar{v}^{\mathrm{T}}\mathbf{p})(\mathbf{p}^{\mathrm{T}}\bar{v}) = \sum \bar{v}^{\mathrm{T}}\mathbf{p}\mathbf{p}^{\mathrm{T}}\bar{v} = \bar{v}^{\mathrm{T}}\sum \mathbf{p}\mathbf{p}^{\mathrm{T}}\bar{v}$$

where $\mathbf{p}^{\mathrm{T}}\mathbf{p} = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$. Comparing with (1) we can get: $\bar{\mathbf{Q}} = \sum \mathbf{p}\mathbf{p}^{\mathrm{T}}$.

In section 3, we present a new method for mesh simplification based on triangle constriction.

## 3 Mesh Simplification Based on Square Volume Error

### 3.1 Square Volume Error with Constraint (SVEC)

**Definitions and notations**: We first introduce some definitions and notations. Given a triangular mesh $M(v,e,f)$.

**Definition 1**: The surrounding triangles which sharing $v$ as the vertex are defined as the adjacent triangles of $v$.

**Definition 2**: The aggregate of adjacent triangles of the three vertices of one triangle $t$ is defined as the adjacent triangles of $t$.

**Definition 3**: The vertices which related with $v$ are defined as the adjacent vertices of $v$.

**Definition 4**: If the number of adjacent triangles of $v$ is equal to the number of adjacent vertices, $v$ is an inner vertex; otherwise $v$ is a boundary vertex and its adjacent triangles are boundary triangles.

The two keys of triangle constriction of mesh simplification are: determine which triangle should be constricted and determine the position of new vertex. How to construct an appropriate and steady error function is an important for the two keys. When mesh simplification, new mesh after triangle constriction will produce volume variation comparing with original mesh. Mesh volume preserving can make the simplification model approximate original model better [12, 14]. The volume variation is defined as the sum of volume of the tetrahedrons whose bottom faces is constricted triangle and its adjacent triangles (see Fig. 2). In Fig. 2, the deletion triangle is $\Delta v_0v_1v_2$, and its adjacent triangles are $\Delta v_0v_3v_4$, $\Delta v_0v_4v_5$, $\Delta v_0v_5v_1$, $\Delta v_1v_5v_6$, $\Delta v_1v_6v_7$, $\Delta v_1v_7v_2$, $\Delta v_2v_7v_8$, $\Delta v_2v_8v_0$, $\Delta v_0v_8v_3$. The new vertex is $\bar{v}$. So after deleting $\Delta v_0v_1v_2$, the volume variation of model is the tetrahedrons whose one of vertices is $\bar{v}$ and bottom faces are the above ten triangles. Let area of triangle $\Delta$ be $S_\Delta$, volume of tetrahedrons whose one of face is $\Delta$ be $V_\Delta$, volume variation of one triangle constriction step be $\sum V$. In Fig. 3, Let coordinates of $\bar{v}$ be $(\bar{x},\bar{y},\bar{z},1)$, plane function of $\Delta v_0v_1v_2$ be $ax+by+cz+d=0$, and $\mathbf{p}=(a,b,c,d)$, then, $V_{\Delta v_0v_1v_2} = \frac{1}{3}(\bar{v}\cdot\mathbf{p}^{\mathrm{T}})\cdot S_{\Delta v_0v_1v_2}$. This is a signed volume. If after triangle constriction steps, volume variation of part of model is positive and part of model is negative, the sum of volume variation is zero, so we can't measure the volume variation of model. In the previous methods, [12, 14] used the absolute value of signed volume as the error metric, but this can make additional computation. In this paper, we use square volume accumulation as error metric, denote as square volume error (SVE) which defined as follows:

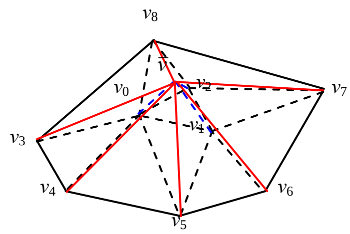


**Fig. 2** Volume variation by triangle constriction

$$V^2_{\Delta v_0v_1v_2} = \frac{1}{9}(\bar{v}\cdot\mathbf{p}^{\mathrm{T}})^2\cdot S^2_{\Delta v_0v_1v_2} = \frac{1}{9}(\bar{v}\cdot\mathbf{p}^{\mathrm{T}}\mathbf{p}\cdot\bar{v}^{\mathrm{T}})\cdot S^2_{\Delta v_0v_1v_2}$$

The sum square volume error by deleting a triangle can be denoted as:

$$\sum V^2 = \sum \frac{1}{9}(\overline{v} \cdot \mathbf{p}^{\mathrm{T}}\mathbf{p} \cdot \overline{v}^{\mathrm{T}}) \cdot S^2 = \overline{v}(\sum \frac{1}{9} S^2 \cdot \mathbf{p}^{\mathrm{T}}\mathbf{p})\overline{v}^{\mathrm{T}} \tag{2}$$

The new vertex $\overline{v}$ after one triangle constriction can be determined by minimizing (2). A temporary matrix $\mathbf{Q}_i$ is defined for each triangle:

$$\mathbf{Q}_i = \frac{1}{9} S_i^2 \cdot \mathbf{p}^{\mathrm{T}}\mathbf{p} = \frac{1}{9} S_i^2 \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix} \tag{3}$$

Square volume error is defined as (2) and (3), constriction error matrix is defined as the sum of the temporary matrixes of constricted triangle and its adjacent triangles: $\overline{\mathbf{Q}} = \sum_{i=1}^{m} \mathbf{Q}_i$ . (1) is used to compute constriction error. For decreasing the maximum error of the square volume variation, we revise the constriction error with a constraint. In this paper, we define the original square volume variation as weighted factors, and all of local square volume variation is weighted as new triangle constriction error:

$$\sum V^2 = \sum \frac{1}{9}(\overline{v} \cdot \mathbf{p}^{\mathrm{T}}\mathbf{p} \cdot \overline{v}^{\mathrm{T}}) \cdot S_i^2 = \overline{v}(\sum \frac{1}{9} \phi_i S_i^2 \cdot \mathbf{p}^{\mathrm{T}}\mathbf{p})\overline{v}^{\mathrm{T}}$$

where $\phi_i$ is the square volume of the tetrahedrons which the vertex is $\overline{v}$ and bottom faces are the triangles on the adjacent region.

3.2 Shape factor and normal constraint factor

When mesh simplification, the bad shaped triangles should be constriction first. For triangular mesh, equilateral triangle is denoted as the best shaped triangle. If one of an internal angle of triangle is near zero degree, the triangle is the worst. Besides, from mesh simplification, triangle with larger area should be better than that with less area. So we can define triangle shape factor as follows:

$$W_{shape\text{-}factor} = 2 \cdot (1 - \cos(\min(\alpha))) \cdot S_\Delta$$

where $\min(\alpha)$ is the least internal angle of one triangle. For $0 < \min\alpha \le \pi/3$, we can get: $0 < 2 \cdot (1 - \cos(\min(\alpha))) \le 1$.

For a given mesh model, the local mesh will be dense at high curvature variation region. So the local square volume error is also small. For avoid constricting triangles which on uneven region, we introduce normal constraint factor. In existing researches, a general method for computing normal is weighted the normal of adjacent triangles of a vertex or a triangle [5]. But this method can produce error sometimes. Such as in Fig. 5, using a 2D example, $f_0$, $f_1$, $f_2$ represent triangles. $n_1$ and $n_2$ are normal of $f_1$ and $f_2$, and the clamp angle of $n_1$ and $n_2$ are near to 180 degree, the weighted of these two normal is near to zero, so the acute region can be estimated as flat region. So we present a new normal constraint factor.

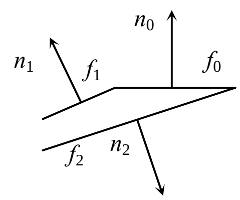



**Fig. 3** Normal computation of meshes in 2D

Normal of a triangle can be computed from its three vertices, let adjacent triangles of $f_0$ are $f_1 \ldots f_m$, and their normal are $n_0 \ldots n_m$, the normal constraint factor of $f_0$ can be defined as:

$$N_0 = \frac{\sum_{i=1}^{m} \left( \frac{|n_0||n_i| - |n_0 \cdot n_i|}{2|n_0||n_i|} \right)}{m}$$

For unit normal vector, above factor is:

$$N_0 = \frac{\sum_{i=1}^{m}(\frac{1-|n_0 \cdot n_i|}{2})}{m}$$

From the definition of normal constraint factor we can see $0 \le N_0 \le 1$. When a triangle and its triangles are on the same plane, $N_0 = 0$. We add the shape factor and normal constraint factor into constriction error, then (4) can be re-defined as follows:

$$\mathbf{Q}_i = \frac{1}{9} W^i{}_{shape-factor} \cdot N^i{}_{normal-factor} \cdot S^2{}_i \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix} \tag{4}$$

Constriction error of each triangle is:

$$\Delta_{error} = \overline{v}(\sum_i \mathbf{Q}_i)\overline{v}^{\mathrm{T}} = \overline{v} \cdot \overline{\mathbf{Q}} \cdot \overline{v}^{\mathrm{T}} \tag{5}$$

From (5) we can see constriction error not only consider the shape and normal of constricted triangle but also all of the triangles on constriction region, so the constriction error is more reasonable. In this paper, we insert all of triangles into a heap list by the constriction cost from small to big. In each constriction step, the triangle with the least cost is constricted first.

3.3 Gauss curvature filter

In mesh simplification, for preserving the detail features of model, the sum of the absolute values of two principle curvature of vertex is defined as feature factor. However, this factor needs to compute the mean curvature and Gauss curvature of vertex, which will improve the complication of mesh simplification. Gauss curvature is an important embedded geometric property, which can be used to describe a vertex's geometric feature. Let $k_1, k_2$ are the two principle curvatures of one vertex *v*, $k = k_1 \cdot k_2$ is the Gauss curvature of *v*. Gauss curvature can reflect the bend degree of surface. In this paper, we use the Gauss curvature estimating method in [15] defined as:

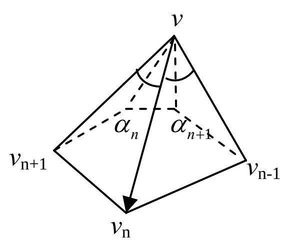



**Fig. 4** Mesh vertex and its adjacent angles

$$K_v = \frac{2\pi - \sum_n \alpha_n}{A_V(v)}$$

where $\alpha_n$ is the adjacent angle of $v$ shown as in Fig. 4, $A_V(v)$ is the area summation of the adjacent triangles of $v$. In this paper, we use the absolute value of Gauss curvature of vertex to define the Gauss curvature of a triangle:

$$K_i = \left|\frac{2\pi - \sum_n \alpha_n}{A_V(v_1)}\right| \cdot \omega_{v_1} + \left|\frac{2\pi - \sum_n \alpha_n}{A_V(v_2)}\right| \cdot \omega_{v_2} + \left|\frac{2\pi - \sum_n \alpha_n}{A_V(v_3)}\right| \cdot \omega_{v_3} \tag{6}$$

where $\omega_{v_j}$ are weights defined as follows:

$$\omega_j = \frac{A_V(v_j)}{A_V(v_1) + A_V(v_2) + A_V(v_3)} \qquad (j = 1, 2, 3)$$

Resetting (7) we have the Gauss curvature of each triangle is:

$$K_i = \frac{\left|2\pi - \sum_n \alpha_{1,n}\right| + \left|2\pi - \sum_n \alpha_{2,n}\right| + \left|2\pi - \sum_n \alpha_{3,n}\right|}{A_V(v_1) + A_V(v_2) + A_V(v_3)}$$

In mesh simplification, if Gauss curvature of one triangle is more than a given threshold is deemed as strong feature triangle, and the vertex with maximal absolute Gauss curvature is the strong feature vertex, so while mesh simplification, the strong feature vertex should keep from deletion first.

3.4 New vertex determination

The existing condition of new vertex is constriction error matrix $\bar{\mathbf{Q}}$ is reversible. Expanding $|\bar{\mathbf{Q}}|$ we have:

$$\bar{q}_{13}(\bar{q}_{12}\bar{q}_{23}-\bar{q}_{22}\bar{q}_{13})-\bar{q}_{23}(\bar{q}_{11}\bar{q}_{23}-\bar{q}_{12}\bar{q}_{13})+\bar{q}_{33}(\bar{q}_{11}\bar{q}_{22}-\bar{q}_{12}\bar{q}_{12})$$

This formula can be treated as the condition that if the constricted triangle and its adjacent triangles are on the same plane or not. If the constricted triangle and its adjacent triangles are coplanar then:

$$\bar{q}_{13}(\bar{q}_{12}\bar{q}_{23}-\bar{q}_{22}\bar{q}_{13})-\bar{q}_{23}(\bar{q}_{11}\bar{q}_{23}-\bar{q}_{12}\bar{q}_{13})+\bar{q}_{33}(\bar{q}_{11}\bar{q}_{22}-\bar{q}_{12}\bar{q}_{12})\neq 0$$

If not then:

$$\bar{q}_{13}(\bar{q}_{12}\bar{q}_{23}-\bar{q}_{22}\bar{q}_{13})-\bar{q}_{23}(\bar{q}_{11}\bar{q}_{23}-\bar{q}_{12}\bar{q}_{13})+\bar{q}_{33}(\bar{q}_{11}\bar{q}_{22}-\bar{q}_{12}\bar{q}_{12})=0$$

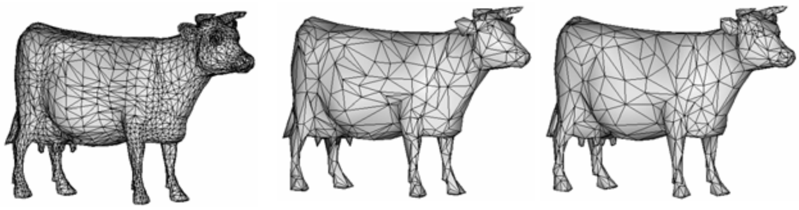
In the latter case, minimizing (1) we can't determine the position of new vertex $\bar{v}$. So we can use a weighted method for determining $\bar{v}:\bar{v}$ is substituted with the weighted of the three vertices of triangle:

$$\bar{v}=L_i v_i+L_j v_j+L_k v_k$$

where $L_i$, $L_j$ and $L_k$ are the adjacent area weights of vertices and $L_i+L_j+L_k=1$.

$$L_i=\frac{A_{v_i}}{A_{v_i}+A_{v_j}+A_{v_k}},\ L_j=\frac{A_{v_j}}{A_{v_i}+A_{v_j}+A_{v_k}},\ L_k=\frac{A_{v_k}}{A_{v_i}+A_{v_j}+A_{v_k}}$$

Then the new vertex is nearer to the barycenter of the adjacent region of constricted triangle, the new triangles can have better shape. From Fig. 5 we can see the details of model can be preserving better with the new method. (a) is the original model, (b) is the simplification result with [8] method, (c) is our method.



**(a)** **(b)** **(c)**

**Fig. 5** Simplification of cow model by 80%

3.5 Boundary triangles processing and error modification

For non-closed model, Boundary triangles can be classified as vertex boundary triangle and edge boundary triangle, if only one vertex is boundary vertex, triangle is denoted as vertex boundary triangle (see as triangle *A* in Fig. 6). If two vertices are boundary vertices, triangle is denoted as edge boundary triangle (see as triangle *B* in Fig. 6). For vertex boundary triangle, new vertex is the boundary vertex of constricted triangle; for edge boundary triangle, new vertex is the weighted of the two boundary vertices:

$$\bar{v}=\omega v_i+(1-\omega)v_j \qquad (0\le\omega\le 1)$$

Substitute above $\bar{v}$ into (1), and let $\partial\Delta(\bar{v})/\partial\omega=0$, we can determine $\omega$, so $\bar{v}$ can be determined.

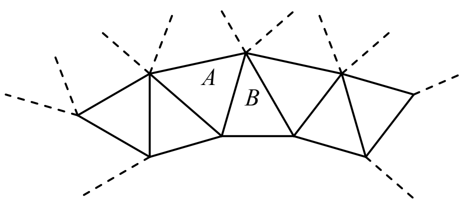


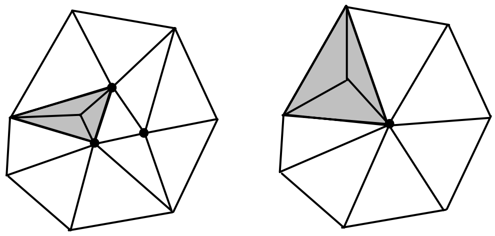

**Fig. 6** Boundary triangles

When a triangle is constricted, the adjacent triangles are also deleted. The new generate triangles should be re-computed the constriction error. When the most triangles are locate on a same plane, the constricting error of each triangle which is coplanar with its adjacent triangles is always zero. So the constricting list can't be determined. In this paper, if the constricting error of new triangles is zero, then we insert the triangles behind of triangles whose constriction error are also zero. Fig. 7 shows an example for a non-closed model which most of triangles are coplanar. (a) is the original model; (b) is the simplification result without re-queuing; (c) is the simplification result with re-queuing. We can see the re-queuing method can achieve better triangles.

**(a)** **(b)** **(c)**

**Fig. 7** Non-closed mesh simplification when triangles are coplanar

After each triangle constriction, we test the new triangles with conditions. Such as in Fig. 8, the shadow region contain a hidden cycle; performing such a triangle constriction would introduce a new cycle, which is undesirable. If the adjacent triangles of local region have a cycle, we combine the three triangles into one triangle.



**(a)** **(b)**

**Fig. 8 (a)** Adjacent triangles have a cycle; **(b)** Adjacent triangles have a hidden cycle

3.6 Algorithm description

Heap sort method is used for sorting the triangles sequence by the constricting cost, and the triangles are saved from high cost to low cost. In each simplification step, the triangle with the least constricting cost will be constricted firstly. A triangle constricting sequence is built in memory which can record the constricted triangles. Then a progressive model can be set up and a simplified model can return to the original model with inverse operation.

**Algorithm Description:**

***Step* 1:** For each triangle of original mesh, a temporary matrix $\mathbf{Q}_i$ can be computed by **Eq.**(5);

***Step* 2:** Computing constricting error matrix $\overline{\mathbf{Q}}$ for each triangle;

***Step* 3:** If a triangle is an inner triangle:

If $\begin{vmatrix} \bar{q}_{11} & \bar{q}_{12} & \bar{q}_{13} & \bar{q}_{14} \\ \bar{q}_{12} & \bar{q}_{22} & \bar{q}_{23} & \bar{q}_{24} \\ \bar{q}_{13} & \bar{q}_{23} & \bar{q}_{33} & \bar{q}_{34} \\ 0 & 0 & 0 & 1 \end{vmatrix} \neq 0$, then the new point's position is computed with $\overline{\mathbf{Q}}$;

If $\begin{vmatrix} \bar{q}_{11} & \bar{q}_{12} & \bar{q}_{13} & \bar{q}_{14} \\ \bar{q}_{12} & \bar{q}_{22} & \bar{q}_{23} & \bar{q}_{24} \\ \bar{q}_{13} & \bar{q}_{23} & \bar{q}_{33} & \bar{q}_{34} \\ 0 & 0 & 0 & 1 \end{vmatrix} = 0$, then the new point's position is computed with Eq.(8);

If a triangle is a boundary triangle, the new point can be generated by the boundary points of this triangle;

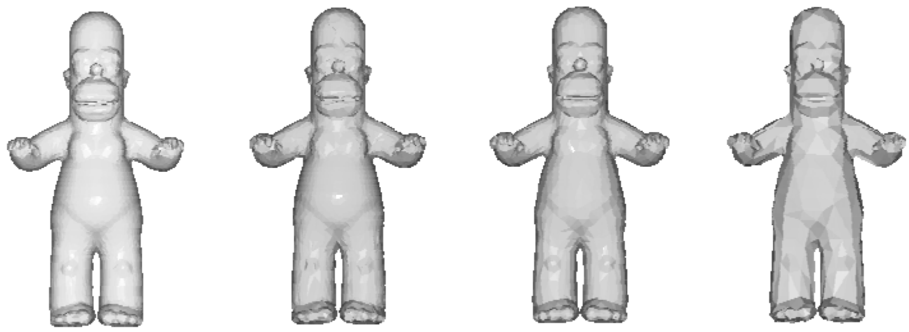
***Step* 4:** Using **Eq.**(6) to compute the constricting error for each triangle, and insert each triangle into the triangle constriction heap queue;

***Step* 5:** ① Extract the top triangle in the heap queue and compute its Gauss curvature factor. Using a given threshold to estimate the triangle is a strong feature triangle or not. If it is, then the strong feature point can be denoted as the new point within constriction process. If it is not, then the new point can be obtained from constriction process.

② Update the temporary matrixes $\mathbf{Q}_i$ of the new triangles, and re-compute the constriction error of the new triangles and the adjacent triangles of them. Insert these triangles into the constriction queue. If the constriction error of one triangle is zero, then insert this triangle into the end of the triangles whose constriction errors are all zero;

***Step* 6:** If arrive the needed simplification ratio, the iteration will be stopped. Otherwise go to ***Step* 5**.

Fig. 9 gives an example for constructing a progressive model by the new method. The simplification ratio of (a), (b), (c) and (d) are 30%, 50%, 70% and 90%, respectively.



**(a)** **(b)** **(c)** **(d)**

**Fig. 9** Progressive model construction

## 4 Experiments

In this section, experiments are all implemented on PC with 2.8GHz Pentium(R) 4 CPU, 1GB memory. Fig. 10 shows an example for mesh simplification. New simplification method, QEM based on edge collapse [3] and QEM based on triangle constriction [8] are used to simplify the mesh model to 70% (the number of triangles after simplifying is 6792) and 95% (the number of triangles after simplifying is 1132). In Fig. 10, (a) is the original model; (b) and (c) are the results using method [3]; (d) and (e) are the results using method [8]; (f) and (g) are the results using new method. From Fig. 11 we can see, the simplification meshes by new method are more uniform. Fig. 12 shows the leg details in Fig. 11 with 70% simplification. (a) is method [3]; (b) is method [8]; (c) is the new method. Table 1 gives the square volume error, *Hausdorff* distance and simplification speed comparison of three methods. The square volume error is defined as:

$$\Delta V = \overline{v} \cdot \overline{\mathbf{Q}} \cdot \overline{v}^{\mathrm{T}} = \overline{v}(\sum \mathbf{Q}_i)\overline{v}^{\mathrm{T}}$$

*Hausdorff* distance error is defined as:

$$Dis_{Hausdorff}(A,B) = \max_{p_A \in A}\{\min_{p_B \in B}\{d(p_A, p_B)\}\}$$

From Table. 1 we can see, simplification mesh based on new method has less square volume error than other two methods. In low simplification ratio, the *Hausdorff* distance error of new method is less than other two methods. Triangle constriction method can delete four triangles in one constriction step. For estimating Gauss curvature, New method is slower than method [8], however, is much faster than method [3].

## 5 Conclusion and Future Work

This paper presents a new mesh simplification method based on triangle constriction. New method combines the square volume error with constraint (SVEC) and two feature factors: geometric shape factor and normal constraint factor to define triangle constricting error which substitute distance error of QEM. The triangle which located adjacent region is flatter, and the constricted triangle with bad shape and the error of square volume is less will be constricted first. Strong feature triangle is judged with Gauss curvature factor, for non-strong feature triangle, new vertex position is determined by minimizing the collapse error of mesh model. For strong feature triangle, new vertex is the strong feature vertex of the three triangle vertices. New method is simple, steady and can preserve mesh feature well. Comparing with existing methods, the simplification mesh based on new method has less visual difference with original model. The further work is using global mesh optimization for mesh simplification.

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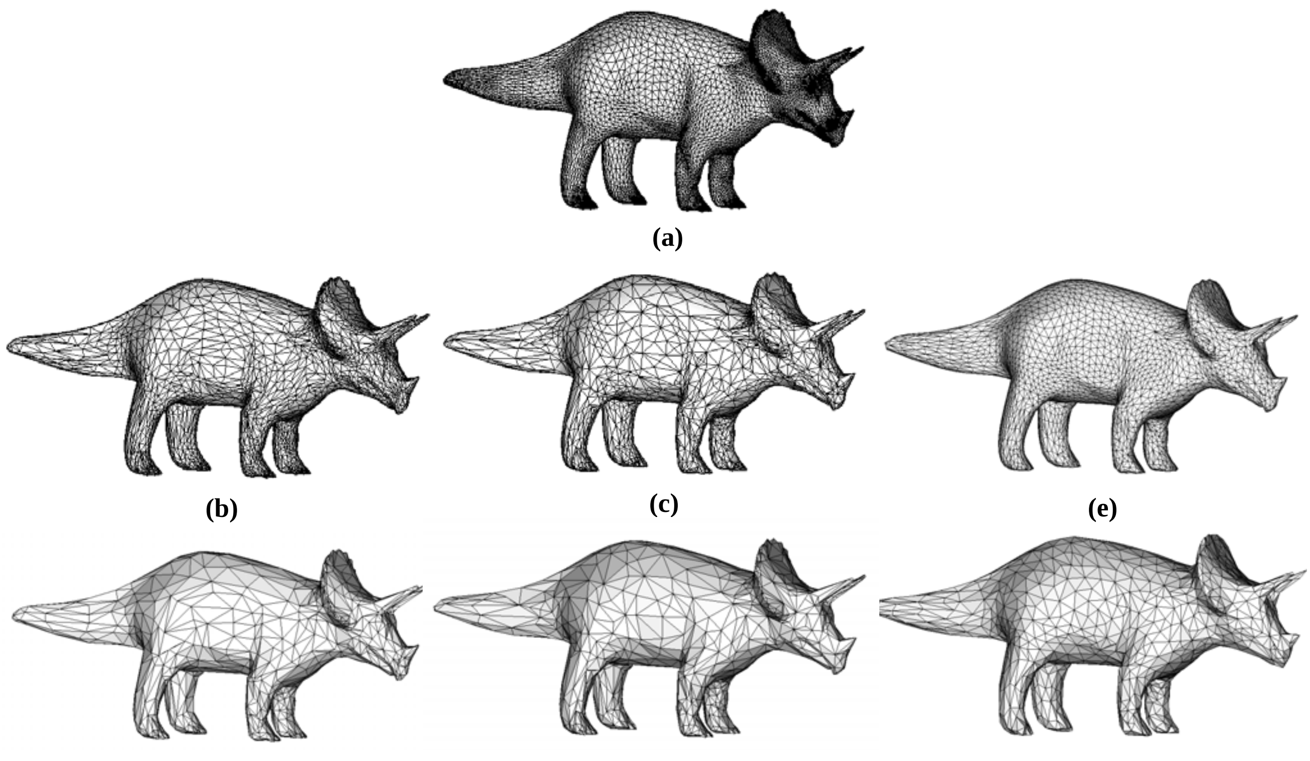
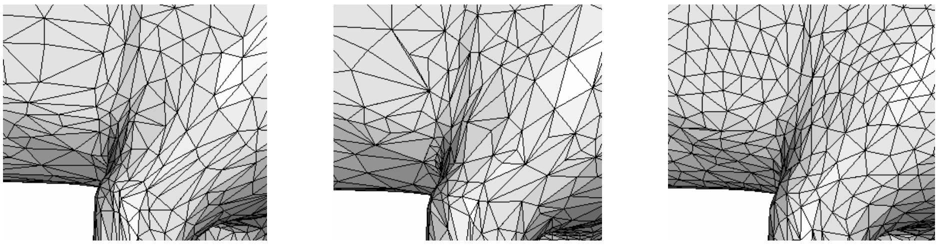


**(a)**

**(b)** **(c)** **(e)**

**(e)** **(f)** **(g)**

**Fig. 10** Mesh simplification Triceratops model (11322 vertices, 22640 triangles)



(a) (b) (c)

**Fig. 11** Model details of Triceratops with three simplification methods

**Table. 1** Simplification error and speed comparison of Triceratops model

| | **SVE (70%)** | **SVE (95%)** | **70% time spending (ms)** |
|---|---|---|---|
| **Method [3]** | 1.157243e-5 | 5.668615e-5 | 547 |
| **Method [8]** | 9.226872e-6 | 2.452733e-5 | 359 |
| **New Method** | 6.127509e-6 | 1.825221e-5 | 375 |
| | ***Hausdorff* distance (70%)** | ***Hausdorff* distance (95%)** | **95% time spending (ms)** |
| **Method [3]** | 1.012878e-3 | 5.823386e-2 | 750 |
| **Method [8]** | 3.229115e-3 | 6.236649e-2 | 453 |
| **New Method** | 2.556391e-3 | 8.732161e-2 | 469 |