Research Proposal

Tessellation, Fairing, Shape Design, and Trimming Techniques for Subdivision Surface based Modeling

Dr. Fuhua (Frank) Cheng

Department of Computer Science
University of Kentucky
Lexington, Kentucky 40506-0046

Tel: (859) 257-6760
Email: cheng@cs.engr.uky.edu
http://www.cs.engr.uky.edu/˜ cheng/

January 25, 2004

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1 Project Summary

The objective of the proposed research is to develop necessary geometric algorithms and design technologies that support subdivision surface based modeling in CAD/CAM applications. We will especially focus on developing *tessellation*, *automatic fairing*, *shape design* and *trimming* techniques for subdivision surfaces. Development of these algorithms/technologies is critical because they are the building blocks of many subdivision surface based modeling operations and, hence, are needed by any of the CAD/CAM systems that intends to include subdivision surfaces as the next generation surface representation for CAD/CAM applications.

Specific aim has been set for each of these four areas. The aim for *surface tessellation* is to develop *error control techniques* and *adaptive refinement techniques* for subdivision surfaces. The error control techniques include sharp, control point based *error predicting techniques* and precision-guaranteed *subdivision level* computation techniques. The adaptive refinement techniques include *label-driven* and *error-driven* tessellation approaches. These techniques allow concurrent, non-uniform tessellation of the control mesh faces while satisfying the crack-free requirement and the convergence process is subdivision scheme related, i.e., one would automatically get type A limit surface if type A subdivision scheme is used in the new node generation process. The aim for *automatic surface fairing* is to develop *fairness indicating techniques* and automatic *surface irregularity detecting and correcting techniques* for subdivision surfaces. Control point based and *shadowgraph line* based fairness indicating models will be constructed. An *indexing scheme* will be developed to accelerate the generation of these models. Local irregularities are removed by requiring the new surface to match the target shape set by the fairness indicating model. The fairing process will be performed with knot refinement and constrained optimization techniques to avoid unnecessary deformation of the subdivision surface. The aim for *shape design* is to develop *interpolation-based shape design techniques* and *control mesh manipulation techniques* for subdivision surfaces. The interpolation-based design techniques include the construction of a subdivision surface to interpolate a given net of parametric curves. The interpolation process will be developed based on existing subdivision schemes to avoid increasing the complexity of the surface intersection process of the CAD system. The manipulation techniques include manipulating control points of the control mesh and direct manipulation of the limit surface. The aim for *surface trimming* is to develop *computation techniques* for intersection curves (*trimming curves*) of two subdivision surfaces, or a subdivision surface and a parametric surface, and *mesh generation techniques* for trimmed subdivision surfaces for applications in finite element analysis (FEM). New trimming curve representation schemes that avoid excessive node increasing problem and new trimming curve computation techniques that enhance the precision and robustness requirements will be developed.

Results of the proposed research will provide significant advancement in subdivision surface based modeling. Results in the first area will not only provide us with a precise and yet efficient way to generate input to the FEM process, but also an opportunity to address the manufacturability issues in the surface development stage. Results in the second area will significantly shorten the subdivision surface fairing process and make CAD more proactive in the product development process for subdivision surfaces. Results in the third area will provide design engineers with effective tools to sculpt a subdivision surface into desired shape that has never been done before and, therefore, increase the productivity of a design engineer dramatically. Results in the fourth area will provide us with the needed core technologies in performing Boolean operations to form more complicated objects and, consequently, make subdivision surface based modeling operations more reliable. Overall, results of the proposed research will provide the design industry with a solution set for most subdivision surface based modeling operations and, consequently, provide the needed support to make subdivision surfaces the next generation surface representation for CAD/CAM applications.

Broader impacts of this project include providing graduate students with cutting-edge, subdivision surface based modeling experience, enriching curriculum development in geometric modeling and CAD/CAM by integrating research results into graduate courses *Computer Aided Geometric Design* and *Freeform Solid Modeling* (subdivision surface representations and subdivision surface based modeling are included in both courses), training of female PhD students (currently one) and minority students recruited through the department's honor program, and technology transfer and publication.

The proposed research will be carried out by the geometric modeling team at the University of Kentucky in three years. The research is expected to lead to two PhD theses, and nine to twelve publications.

2 Results from prior NSF support (PI: F. Cheng)

Award Number: DMI-9400823, **Amount:**$270,000, **Duration:**8/15/94-7/31/97
Project Title: New Computation Techniques for Shape Modeling and Design
Summary of Completed Work: New techniques in three areas (interproximation, even-degree B-spline interpolation, and automatic blending) of shape design/modeling have been developed. Results in the first area include: (1) new knot choosing technique, (2) interpolating an arbitrary network of curves, (3) interproximating the vertices of an arbitrary mesh, and (4) constrained interpolation. In the second area we have developed a new approach to perform quadratic B-spline curve interpolation at the parameter values where the maxima of quadratic B-spline basis functions occur. The work that has been done in the third area includes: (1) blending arbitrary closed polyhedra, and (2) blending smoothing arbitrary closed polyhedra. In both cases, Varady patches are used in the blending process and the polyhedra can be regular or irregular. (1 Post Doc, 1 Ph.D., 2 MS completed; 5 journal and 5 conference papers published)

Award Number: INT-9722728, **Amount:** $102,522, **Duration:** 6/1/97-5/31/00
Project Title: The GEO-WEB-CAD project
Summary of Completed Work: The goal is to develop network CAD/CAM interfaces and collaborative design paradigms for the following components of an object-oriented geometric modeling tool kit: (1) an open architecture framework for wire frame, surface, solid and feature modeling from a common unified data and file structure, (2) linear, quadric and NURBS geometry modeling, supporting manifold and nonmanifold topologies, (3) realizing Boolean operations, local operations (fillet, rounding, section, extending), Euler operations, and undo/redo operations, and (4) ray tracing and radiosity rendering techniques. Requirements and properties of a collaborative CAD system from the system architecture point of view have been analyzed and a conceptual model that fulfills these requirements has been developed. Two prototype systems have been implemented. One is based on X-Windows System for the UNIX platform. The other one is implemented using JAVA which does not have platform restriction. (1 Post Doc, 4 MS and 8 BS students completed; 6 journal and 3 conference papers published)

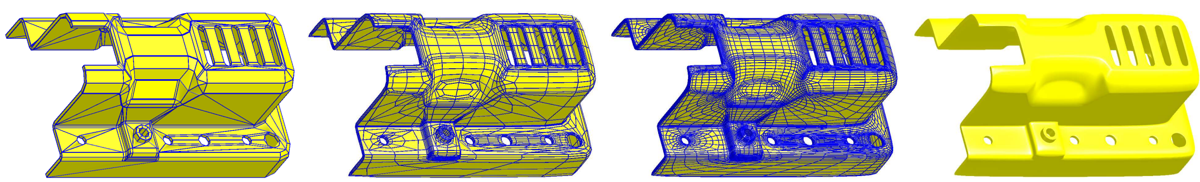
Award Number: DMI-9912069, **Amount:** $393,067, **Duration:** 5/1/00-4/30/04
Project Title: Constrained Design, Streamline Modeling, Automatic Fairing and Automatic Joining Techniques for Non-Uniform Rational B-Spline (NURBS) Surfaces
Summary of Completed Work: The goal is aiming at developing new and efficient techniques that are capable of (1) addressing the manufacturability issues in the surface development stage, (2) providing design engineers with an interactive surface design technique that allows direct manipulation of surface curvature and variation of curvature, and (3) significantly shortening the energy based surface fairing and joining processes and, consequently, make CAD more effective in the product development process. Results that have been achieved include (1) creating G^1/C^1 curves with prescribed shape properties, (2) performing real-time surface design by manipulating tangent vectors instead of control points, (3) detecting and removing surface irregularities using an energy based fairness model of the given surface, and (4) performing smooth, indirect joining. (1 Post Doc and two MS students completed in 2003; 1 Ph.D and 1 MS students will complete in 2004; 6 journal and 4 conference papers published, 2 journal papers accepted, 1 submitted, and 1 in preparation)

Award Number: DMS-0310645, **Amount:** $99,941, **Duration:** 7/1/03-6/30/05
Project Title: Subdivision Surface based One-Piece Representation
Summary of Completed Work: Work on this project is currently in progress. The goal is to develop mathematical theories and geometric algorithms to support subdivision surface based *one-piece representation*, i.e., representing the final object in a modeling process with only one subdivision surface. Results that have been achieved include (1) energy computation techniques for Catmull-Clark subdivision surfaces, (2) efficient evaluation techniques for general subdivision surfaces. (1 Ph.D, and 1 MS students will complete in 2005; 2 papers in preparation)

3 Project Description

3.1 Motivation

Subdivision surfaces have become popular recently in CAD/CAM because of their *flexibility*, *numerical stability*, *simplicity in coding* and, most importantly, their capability in *modeling/representing complex shape of arbitrary topology* [17]. Given a control mesh and a set of *mesh refining rules* (or, more intuitively, *corner cutting rules*), one gets a *limit surface* by recursively cutting off corners of the control mesh [9][19]. The limit surface is called a *subdivision surface* because the corner cutting (mesh refining) process is a generalization of the uniform B-spline surface *subdivision technique*. Therefore, subdivision surfaces include uniform B-spline surfaces and piecewise Bézier surfaces as special cases. It is also recently known that subdivision surfaces include non-uniform B-spline surfaces and NURBS surfaces as special cases [53]. Subdivision surfaces can model/represent complex shape of arbitrary topology because there is no limit on the shape and topology of the control mesh of a subdivision surface [17]. See Figure 1(d) for the representation of a ventilation control component with a single subdivision surface. The initial control mesh of the surface and the control mesh after one refinement and two refinements are shown in (a), (b) and (c), respectively. The ventilation control component has seventeen holes (handles). Therefore, it can not be represented by a single B-spline or NURBS surface. Unfortunately, subdivision surfaces have not received much attention from CAD/CAM industries



(a) (b) (c) (d)

Figure 1: (a) Initial control mesh, (b) control mesh after one refinement, (c) after two refinements, and (d) limit surface of a ventilation control component.

until recently because of two reasons. First, it was not known until 1998 that subdivision surfaces can be parametrized [54]. Without a parametric representation, it is essentially impossible for a CAD/CAM system to include subdivision surfaces as a free-form surface modeling tool because of problems with standard operations such as picking, rendering and texture mapping [54]. The second problem is with hardware. Subdivision surfaces are typically generated through recursive meshing. The complexity of the meshing process grows exponentially with respect to the recursive subdivision level. This made generation and rendering of subdivision surfaces on an ordinary workstation essentially impossible in the 80s and early 90s because of lacking enough memory for the recursive mesh refining process.

Things have changed over the past few years. With the parametrization technique of subdivision surfaces becoming available [54] and with the fact that non-uniform B-spline and NURBS surfaces are special cases of subdivision surfaces becoming known [53], we now know that subdivision surfaces cover both *parametric forms* and *discrete forms*. Since parametric forms are good for design and representation and discrete forms are good for machining and tessellation (including FE mesh generation) [1], we finally have a representation scheme good for all CAD/CAM applications. With powerful PCs that carry almost unlimited memory available everywhere, computation and rendering of subdivision surfaces are no longer a problem either. The era of subdivision surfaces is finally here. Actually, subdivision surfaces have already been used as primitives in several commercial systems such as Alias|Wavefront's *Maya*, Pixar's *Renderman*, Nichiman's *Mirai*, and Microspace' *Lightwave 3D* [7]. They are not used as a major surface representation in CAD/CAM systems yet because of lacking necessary geometric algorithms and modeling techniques in *surface tessellation*, *surface fairing*, *shape design* and *surface trimming*. Development of algorithms and technologies in these areas is important because they are the building blocks of many subdivision surface based modeling operations and, hence, are needed by any of the CAD/CAM systems that intends to include subdivision surfaces as the next generation surface representation for CAD/CAM applications. In the following, we will formally define these problems and review related works.

3.2 Background

3.2.1 A Brief History

The concept of generating a surface through mesh refinement has its root in a curve generation technique developed by Chaikin [10]. In his approach, a curve is generated by recursively cutting off corners of a given polygon. Each recursive cutting cycle generates two new points on each leg of the polygon. If there are $n+1$ vertices $\mathbf{P}_i^j$, $i = 0, 1, ..., n$, after the jth recursive cutting cycle, then the two new points generated on the polygon leg $\mathbf{P}_i^j \mathbf{P}_{i+1}^j$ are defined as follows:

$$\mathbf{P}_{2i}^{j+1} = \frac{3}{4}\mathbf{P}_i^j + \frac{1}{4}\mathbf{P}_{i+1}^j; \qquad \mathbf{P}_{2i+1}^{j+1} = \frac{1}{4}\mathbf{P}_i^j + \frac{3}{4}\mathbf{P}_{i+1}^j.$$

This process generates a uniform, quadratic B-spline curve as this corner-cutting process is nothing but the quadratic B-spine subdivision process. The concept of B-spline subdivision is actually a generalization of Chaikin's algorithm (see [51] for the corresponding refinement equation).

Following Chaikin's work, a variety of subdivision schemes for curves and surfaces have been proposed during the past two decades. For instance, a 4-point subdivision scheme proposed by Dyn, Levin and Gregory [21] can generate a subdivision curve to interpolate given data points. New points for each leg of the refined control polygon are defined by

$$\mathbf{P}_{2i}^{j+1} = \mathbf{P}_i^j; \qquad \mathbf{P}_{2i+1}^{j+1} = \frac{8+\omega}{16}(\mathbf{P}_i^j + \mathbf{P}_{i+1}^j) - \frac{\omega}{16}(\mathbf{P}_{i-1}^j + \mathbf{P}_{i+2}^j)$$

where $0 < \omega < 2(\sqrt{5} - 1)$, to ensure convergence of the refined mesh. The standard value is $\omega = 1$ which has an order three precision.

Refining (subdivision) schemes for subdivision surfaces can be classified into two categories: (1) *approximating techniques*, and (2) *interpolating techniques*. Two typical subdivision schemes in the first category are Doo and Sabin's scheme [20] and Catmull and Clark's scheme [9]. Doo and Sabin's scheme generates a surface by recursively cutting off corners and edges of a given **rectangular mesh** as follows:

1. For every vertex V_i of the current mesh P, a new vertex V_i', called an *image*, is generated on each face adjacent to V_i.
2. For each face F_i of P, a new face, called an *F-face*, is constructed by connecting the *image* vertices V_i's generated in Step 1.
3. For each edge E_i common to two faces F_i and F_i', a new 4-sided face, called an *E-face*, is constructed by connecting the *images* of the end vertices of E_i on the faces F_i and F_i'.
4. For each vertex V_i, where n faces meet, a new face, called a *V-face*, is constructed by connecting the images of V_i on the faces meeting at V_i.

This subdivision scheme generates a uniform biquadratic B-spline surface. Catmull and Clark's scheme [9] is similar to the Doo-Sabin scheme, but is based on tensor product bicubic B-spline. The surface generated by this scheme is C^2 continuous everywhere except at some extraordinary points where it is C^1 continuous. Catmull and Clark's scheme can work on meshes of arbitrary topology. Loop [43] has presented a similar subdivision scheme based on generalization of quartic three-direction Box-splines for triangular meshes. Peters and Reif [47] and Habib and Warren [27] independently introduced schemes that generalize quadratic 4-direction Box Splines on irregualr meshes. Subdivision schemes that can generate surfaces with sharp features [17] or fractionally sharp features [29] have also been proposed. Recently, it is even possible to generate features such as cusps, creases, and darts through the introduction of non-uniform subdivision surfaces [53]. A new subdivision scheme that can produce triangular meshes with small number of vertices is proposed by Kobbelt [37].

The first interpolating scheme for subdivision surfaces was presented by Dyn, Levin and Gregory [22]. The technique, called a *butterfly scheme*, requires a topologically regular setting of the initial (control) mesh to produce a C^1 limit surface. Zorin *et al* [63] and Kobbelt [33] have both developed improved interpolating schemes recently. Kobbelt's scheme is a simple extension of the 4-point interpolating subdivision [21]. Zorin *et al*'s scheme retains the simplicity of the butterfly scheme and results in much smoother surfaces even from irregular initial meshes. These interpolating subdivision schemes also find applications in wavelets on manifolds, multiresolution decomposition of polyhedral surfaces, and multiresolution editing.

Some of the mathematical properties of subdivision surfaces have been studied before. For instance, Doo and Sabin have studied the smoothness behavior of their subdivision surfaces through Fourier transformations and eigen-value analysis of the subdivision matrix [19]. Ball and Storry [3][4] and Reif [50] extended Doo and Sabin' work by deriving various necessary and sufficient smoothness conditions for different subdivision schemes. Specific subdivision schemes have also been analyzed by several other people [16][32][34][52][64]. Nevertheless, most of the geometric algorithms and modeling technologies required in subdivision surface based modeling operations are not well studied yet. Four of these areas are especially critical to the design community.

3.2.2 Surface Tessellation

Given a surface, a major concern in both finite element analysis (FEM) and surface rendering is the generation of an approximating mesh of the given surface (within a given error tolerance) with as few nodes as possible. The approximating mesh is used to analyze the physical performance of the surface or in the scan conversion process of the surface. Smaller number of nodes in the approximating mesh is preferred because it makes the analysis process and the rendering process both more efficient. This process of generating an approximating mesh for a given surface, called *surface tessellation*, has been extensively studied for parametric surfaces [13][44]. It has not been well studied for subdivision surfaces yet.

To generate a good approximating mesh for a subdivision surface, one needs to be able to (1) estimate the error between the control mesh (or, an approximating mesh) and the limit (subdivision) surface, (2) determine the level (depth) of recursive subdivision needed to reach a required precision, and (3) adaptively tessellate the faces of the initial control mesh so that an approximating mesh that is just good enough for the specified precision and yet satisfying the crack-free requirement can be constructed. Existing subdivision schemes can not be used directly in the tessellation process because they lack the so-called *adaptive capability*; they would subdivide all the faces of a mesh even if only one of them does not satisfy the precision requirement and, consequently, would generate approximating meshes with too many nodes (see Figure 2(c) for excessively generated nodes in flat regions of a rocker arm with only two levels of subdivision).

The first adaptive scheme for subdivision surfaces is proposed by Kobbelt [33] for Catmull-Clark subdivision surfaces. The method is performed on a trial-and-error basis and only works for the so-called *balanced nets* which, in addition, have to satisfy some other constraints such as *even critical edges*. A few more general schemes appeared recently for *interpolatory* $\sqrt{3}$*-subdivision surfaces* [38], $\sqrt{3}$*-subdivision surfaces* [37], and *modified butterfly subdivision surfaces* [11]. But they work for triangular control meshes only. Another problem with all the above adaptive schemes is that none of them use the error criterion most commonly used in mechanical part design, i.e., the error between the approximating mesh and the limit surface.

We have worked in all these three areas: *error estimation* [15], *subdivision level (depth) computation* [14], and *adaptive mesh generation* [13][44]. However, the techniques developed for B-spline and NURBS surfaces can not be used for subdivision surface directly because the parameter space of a subdivision surface in general is not rectangular or triangular; it can be of any shape. New techniques have to be developed for each of these areas.

3.2.3 Automatic Fairing

Automatic fairing refers to the process of detecting and removing local irregularities of a surface automatically. Curvature plots have been frequently used to analyze the quality of a surface. Commonly used curvature measures include *Gaussian*, *mean*, and *principal curvatures* as well as *normal curvatures* along given directions. *Isophotes* [49], *reflection lines* [30, 31] and, more recently, *highlight lines* [6, 59] have also been used in assessing the quality of a surface. These techniques prove to be more effective and are becoming more popular recently, especially in automotive body surface design, because they are more intuitive to understand and easier to compute. The smoothness of a surface is measured using indicators such as parametric or geometric continuity.

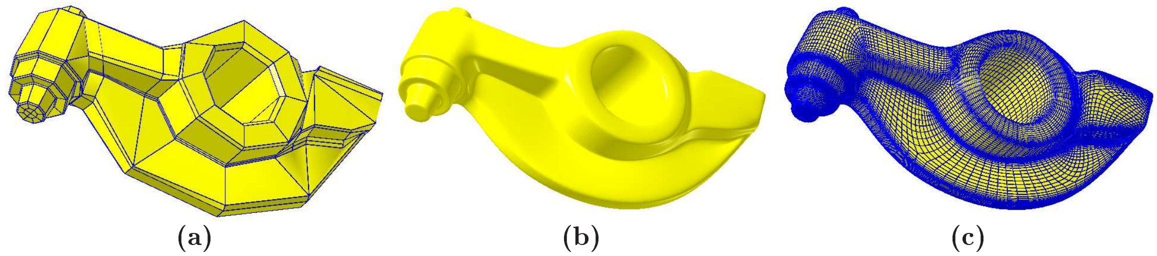


Figure 2: **(a)** Control mesh, **(b)** limit surface and **(c)** an approximating finite-element mesh of a rocker arm.

Several papers analyzing parametric and geometric continuity of subdivision surfaces have been published (see, e.g., [2, 3, 50]). They all concentrate on analyzing the subdivision scheme, instead of the layout of the control points, of the subdivision surface. The latter is actually more important because a well-designed control point net is likely to bring out a higher order of continuity.

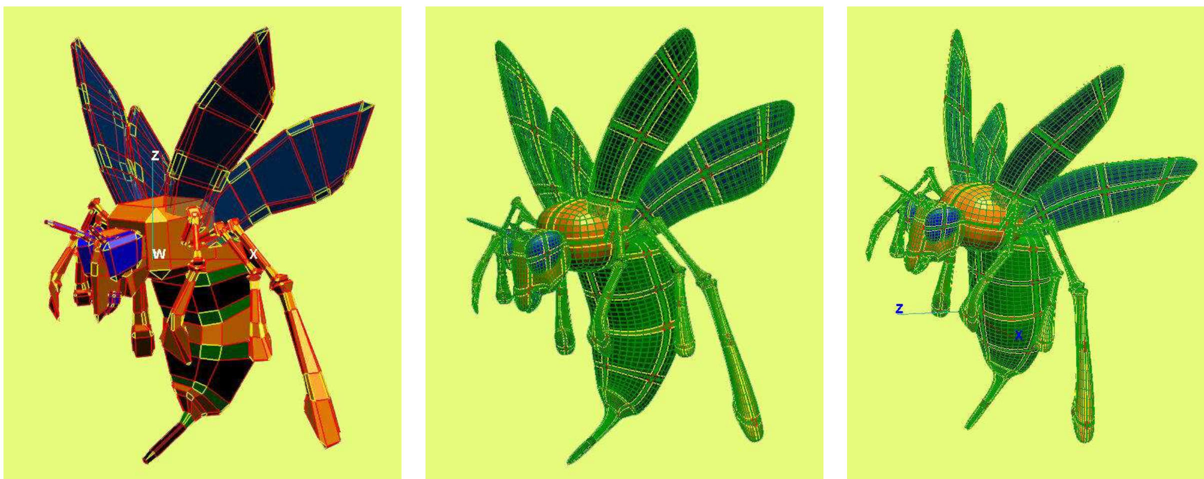
Using diffusion and curvature flow, Desbrun, Meyer, Schröder and Barr [18] have presented a method for removing undesirable noises and uneven edges from irregularly triangulated data. A problem with this approach is that while removing vertices and edges, one might also remove important data "underneath" the "noises". For instance, the "noises" could be introduced by numerical error in the input phase but are within the tolerance level, therefore, the information carried underneath the noises should still be acceptable. A better approach would be to perturb the points or edges to achieve the goal of shape fairing, instead of removing points or edges. However, no paper has been published on constructing a new limit (subdivision) surface with higher parametric or geometric smoothness but with minimum distance from the original limit (subdivision) surface.

Fairing techniques based on modifying reflection or highlight lines have also been proposed [12][30][31][60]. They all heavily rely on the designers to visually identify the irregular regions and to fix them manually by correcting the control points of the surface. This is an experience-based, trial-and-error, and time-consuming process. The complexity of the problem for subdivision surfaces would make the situation even worse, likely to exceed what the human being can cope with, because the topology of a subdivision space is usually much more complicated than that of a parametric surface. One needs the capability of automatic detection and correction of local irregularities for subdivision surfaces. One also needs an approach different from the highlight line model because identifying surface normals that intersect the light source for a subdivision surface is too costly a process for an interactive design environment. A newly developed surface smoothness evaluation model by us, called the *shadowgraph line model*, will be considered here. This model has an analytical representation for each shadowgraph line. Therefore, there is no cost in getting a representation for a shadowgraph line at all.

3.2.4 Shape design

The design of a subdivision surface involves (1) the design, and (2) fine tuning of the control mesh. The only known technique in the first area is the work of Levin [41] which uses a combined subdivision scheme to construct a subdivision surface to interpolate a given net of curves. This is an important work because it points out a better approach for subdivision surface shape design (a parallel work for parametric surfaces can be found in [57]). However, properties of Levin's surface are not known yet and it is not a good idea to include too many new subdivision schemes in a modeling system. It is preferred to have similar interpolation techniques using existing subdivision schemes so that the trimming process can be handled with efficiency (see next section for the justification).

The only known technique in the second area is the work by Miura, Wang and Cheng [45] which provides the user with a tangent manipulation technique to fine tune the shape of a subdivision surface. An example is shown in Figure 3 where a set of Doo-Sabin surfaces are deformed using the tangent vector blending technique and the resulting Doo-Sabin surfaces in non-uniform form are shown in (b). For comparison purpose, the original Doo-Sabin surfaces in non-uniform form are shown in (c). The advantage of this approach is that through the manipulation of the tangent vectors, one can directly manipulate the curvature and variation of curvature of the surface. The disadvantage is that it could be too laborious for subdivision surfaces with complex topology. Note that while it is necessary to provide the user with the capability of direct control point or tangent vector manipulation, it is essential that the user can manipulate the shape of the surface directly (such as dragging a point of the surface to a new location), leaving the time-consuming job of finding the new locations of the control points to the system, so that the fine tuning process of shape design can be carried out more efficiently.



(a) **(b)** **(c)**

Figure 3: (a) Corresponding control mesh, (b) fine tuned Doo-Sabin surfaces in non-uniform form, (c) original Doo-Sabin surfaces in non-uniform form.

3.2.5 Surface Trimming

NURBS surface intersection, even up to today, is still considered the most difficult problem and one of the weaker links in even high end commercial CAD systems [42][55]. The subdivision surface intersection problem would be even more difficult because of the irregularity of the topology of a subdivision surface. The main difficulty is the development of a reliable and efficient computation (marching) process.

An algorithm for calculating the trimming curves of two Loop's subdivision surfaces is proposed by Litke, Levin and Schröeder [42] recently. The algorithm can guarantee exact interpolation of the trimming curves. This is achieved by introducing a new type of surfaces, called *combined surfaces*, to approximate the trimmed surfaces. A problem with this approach is that the inclusion of a new surface type in a CAD system with m surface representation schemes requires m more functions to implement the surface intersection problem. It is preferred to keep the number of surface representation schemes low in a CAD system.

Biermann, Kristjansson and Zorin [8] have presented a new method to approximate Boolean operations on free-form solids. The result of a Boolean operation is approximated by a multiresolution surface. The work pays more attention to efficiency and robustness than to precision and, consequently, is more suitable for applications where precision modeling is not required, such as animation. For applications in CAD/CAM, however, one needs to pay more attention to precision and robustness than to efficiency.

3.3 Objective

The objective of the proposed research is to develop necessary geometric algorithms and design technologies that support subdivision surface based modeling in CAD/CAM applications. We will focus especially on developing *tessellation*, *automatic fairing*, *shape design* and *trimming* techniques for subdivision surfaces. Specific aim has been set for each of these four areas. The scope of the proposed research is illustrated in Figure 4 where a block or a processor bounded by dotted line indicates that block or processor will be built with known technology. Development of the proposed technologies for subdivision surfaces is important because

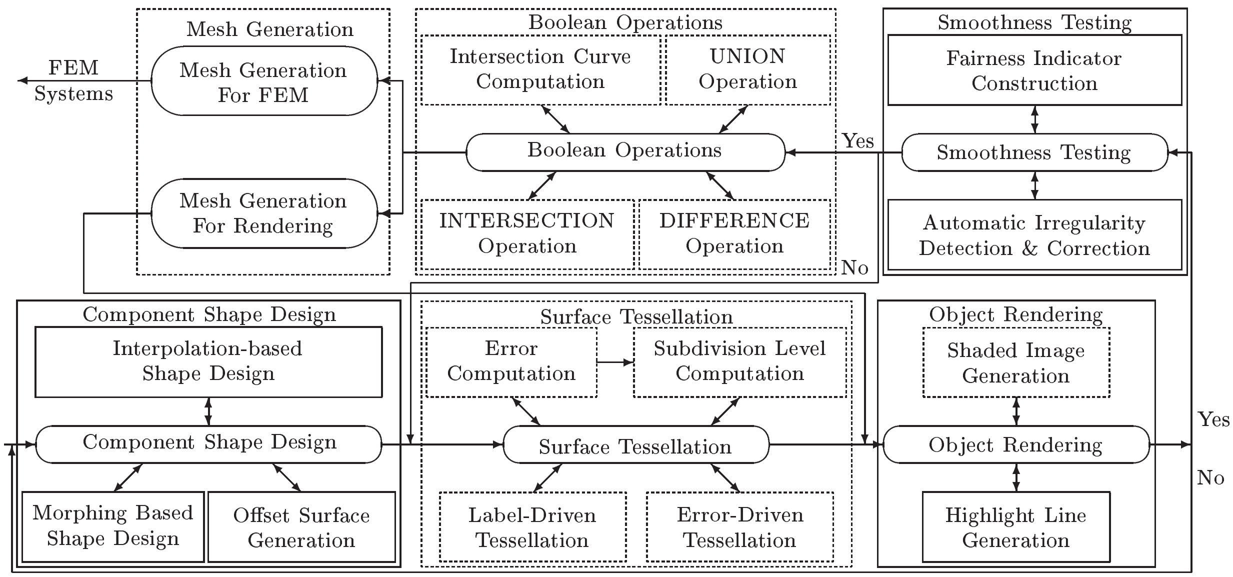


Figure 4: A subdivision surface modeling system with modules on tessellation, fairing, shape design, rendering, Boolean operations and mesh generation.

they are the building blocks of many subdivision surface based modeling operations and, hence, are needed by any of the CAD/CAM systems which intends to include subdivision surfaces as the next generation surface representation for CAD/CAM applications.

(a) Surface Tessellation: The aim for this area is to develop *error control techniques* and *adaptive refinement techniques* for subdivision surfaces. The error control techniques include a control point based *numerical formula* for estimating the error between the control mesh and the limit surface, and an error-tolerance driven *subdivision level* computation technique for the construction of approximating meshes of the limit surface. The subdivision level computation technique guarantees that, after the required recursive subdivision, the resulting approximating mesh is within the given error tolerance of the limit surface. The adaptive refinement techniques include a *label-driven tessellation technique* and an *error-driven tessellation technique*. The label-driven tessellation technique allows parallel tessellation of the control mesh faces without the possibility of generating cracks between adjacent faces. The error-driven tessellation process tessellates a face only when it is necessary for the error tolerance requirement or the crack-free requirement. Any existing subdivision scheme can be used to generate new nodes in the tessellation process and the resulting mesh converges to the limit surface of the utilized subdivision scheme automatically.

(b) Automatic Surface Fairing: The aim for this area is to develop *fairness indicating techniques* and *automatic surface irregularity detecting and correcting techniques* for subdivision surfaces. The fairness indicating techniques include a control point based approach and a *shadowgraph line* based approach. *Fairness indicating models* based on these techniques will be built and an *indexing scheme* will be used to accelerate the generation of these models. Detected local irregularities of a subdivision surface are removed by adjusting its control points so the new surface would match the target shape in those regions set by the fairness indicating model. The fairing process will be performed with knot refinement and constrained optimization techniques to avoid unnecessary deformation of the subdivision surface. This will be an important breakthrough in subdivision surface fairing because adjusting control points of a subdivision surface to improve its smoothness is an unexpectedly difficult process due to both the complexity of its topology and the complex layout of its control points.

(c) Shape Design: The aim for this area is to develop *interpolation-based shape design techniques* and *control* *mesh manipulation techniques* for subdivision surfaces. The interpolation-based design techniques include the construction of a subdivision surface to interpolate a given net of parametric curves. The interpolation process is based on existing subdivision schemes to avoid creating new subdivision schemes and, consequently, avoid increasing the complexity of the surface intersection process of the geometric modeling system. The manipulation techniques include manipulating control points of the control mesh and direct manipulation of the limit surface.

(d) Surface Trimming: The aim for this area is to develop *computation techniques for intersection curves* (*trimming curves*) of two subdivision surfaces, or a subdivision surface and a parametric surface, and *mesh generation techniques* for trimmed subdivision surfaces for applications in finite element analysis (FEM) and surface rendering. New trimming curve representation scheme that avoids excessive node increasing problem and new trimming curve computation technique that enhances the precision and robustness requirements will be developed. The mesh generation techniques for FEM application and for rendering application will accommodate the different requirements for these applications.

3.4 Approaches

3.4.1 Surface Tessellation

Error Estimation and Subdivision Level Calculation: The first task is to estimate the error between the control mesh (or, an approximating mesh) and the limit surface for a given subdivision scheme. Subdivision surfaces would be more widely used, especially in mechanical part design, if one knows how to estimate the error between a part designed with subdivision surfaces and its approximating mesh. The second task here is to estimate the level of recursive subdivision that has to be performed on a face of the control mesh so the resulting mesh would be close enough to the limit surface (within a given error tolerance). This technique enables one to estimate the size of an approximating mesh with required precision (such as the number of nodes) before it is constructed.

Two approaches will be considered for the first task. The first one is to consider a subdivision surface as a group of components whose parameter spaces are rectangular or triangular. The error in this case can be computed as $E = \max_{\forall j}\{E_j\}$ where E_j is the error of the jth component. It is sufficient to consider the case that the parameter space of a subdivision surface is composed of rectangular components only. Suppose that each component is a C^2 parametric surface $\mathbf{S}_j(u,v)$ with parameter space $\mathbf{D}_j = [0,1]\times[0,1]$, then E_j can be estimated using a different version of Filip-Magedson-Markot's formula [24],

$$E_j \le \frac{M_1 + 2M_2 + M_3}{8}, \tag{1}$$

for interpolating subdivision schemes, and the following new formula

$$E_j \le \frac{M_1 + 2M_2 + M_3}{8} + \max\{D_{0,0}, D_{0,1}, D_{1,0}, D_{1,1}\} \tag{2}$$

for other subdivision schemes , where

$$M_1 = \sup_{(u,v)\in\mathbf{D}_j}\left\|\frac{\partial^2\mathbf{S}_j(u,v)}{\partial^2 u}\right\|; \qquad M_2 = \sup_{(u,v)\in\mathbf{D}_j}\left\|\frac{\partial^2\mathbf{S}_j(u,v)}{\partial u\partial v}\right\|; \qquad M_3 = \sup_{(u,v)\in\mathbf{D}_j}\left\|\frac{\partial^2\mathbf{S}_j(u,v)}{\partial^2 v}\right\|,$$

with $D_{0,0} = \|\mathbf{S}_j(0,0)-\mathbf{P}_{0,0}\|$, $D_{0,1} = \|\mathbf{S}_j(0,1)-\mathbf{P}_{0,1}\|$, $D_{1,0} = \|\mathbf{S}_j(1,0)-\mathbf{P}_{1,0}\|$, $D_{1,1} = \|\mathbf{S}_j(1,1)-\mathbf{P}_{1,1}\|$, and $\mathbf{P}_{k,l}$ $(k,l=0,1)$ are vertices of the control mesh component. To achieve a tighter bound than formula (1) or (2), we need to consider more details of the subdivision scheme in the formulation process. Sometimes, it is preferred to have an error formula expressed in terms of control points directly so that it can be easily understood and used. We will analyze the relationship between the bounds of derivatives of the subdivision surface and its control points to get such a formula as well.

In the second approach, the subdivision scheme will be analyzed directly. For each recursive subdivision step, new vertices can be obtained from old ones through vertex-matrix multiplication : $\mathbf{V}_{k+1} = \mathbf{M}\mathbf{V}_k$. Therefore, another possibility in developing the error formula is to estimate this equation directly.

For subdivision level calculation, there are also two possible approaches. The first one consists of two steps. First, compute a bound on the lengths of the face edges which guarantees closeness of a face to the limit surface (within the given error tolerance) if edges of the face are smaller than the bound. Then, analyze the relationship between the bound of the face edges and the subdivision level so that after performing the required subdivision, all the edges of the resulting mesh faces would be smaller than the bound. The second approach is to obtain the subdivision level from the above error estimating process directly. Once the error estimating formula is available, the relationship between the subdivision level and the error is studied so that a formula can be developed.

Label-Driven Tessellation: Given an initial control mesh, a subdivision scheme, an error tolerance, and a subdivision level for each face of the mesh, a parallel method for the construction of an approximating mesh of the limit surface will be developed. The approximating mesh is within the given error tolerance of the limit surface, but has much fewer nodes than the approximating mesh constructed using the conventional approach. The new approach, which works on individual faces recursively, satisfies the following requirements:

- New nodes are generated using the given subdivision scheme and the resulting mesh converges to the limit surface of the given subdivision scheme. If a different subdivision scheme is used in the new node generation process, then the resulting mesh would converge to the limit surface of that subdivision scheme.
- The resulting mesh is *crack-free*.

The basic idea is to use the *unbalanced subdivision scheme* proposed in [13], coupled with the *balanced subdivision scheme*, to overcome the problem of having different subdivision levels for different adjacent faces. A *label* will be assigned to each vertex of the control mesh first. The tessellation process of a mesh face is driven by the labels of its vertices. Vertices generated on the resulting mesh of a mesh face depend on the labels of its vertices only. The label assignment scheme follows the principle that the label assigned to a vertex should be as small as possible, but should ensure the *flatness* of the resulting mesh (flat within the error tolerance of the limit surface). This follows from the observation that smaller vertex labels produce fewer nodes in the resulting approximating mesh.

The tessellation process needs to determine, after each subdivision cycle, which nodes should be taken from the previous level and which nodes should be generated using the given subdivision scheme. It needs to ensure that all the nodes required in the computation of the new nodes are available for the computation process. A node is removed only if it is not used as a new node or it is no longer needed in the computation process of the new nodes.

Error-Driven Tessellation: Another approach for adaptive refinement is to drive the tessellation process by error between the current mesh and the limit surface. For each recursive subdivision cycle, those faces that are not within the given error tolerance of the limit surface are subdivided one more time, and the other faces are subdivided only if it is needed to avoid *cracks*. This approach will in general produce fewer nodes than the approach proposed above.

After the error checking process, labels will be assigned to faces of the new mesh as follows:

$$l_f(f) = \begin{cases} 1, & \text{if face } f \text{ is not within the given error tolerance of the limit surface} \\ 0, & \text{otherwise} \end{cases} .$$

Labels are then assigned to the vertices of the new mesh, subject to the following requirements:

- Labels assigned to the vertices should be as small as possible (to keep the number of nodes in the resulting mesh low).
- Labels assigned to the vertices should guarantee a crack-free status of the resulting mesh.
- Labels assigned to the vertices should guarantee the validity of the new node computation process for the mesh.

These requirements, especially the last one, are critical because vertices are in general not in the same subdivision level after one or more recursive subdivision cycles.

3.4.2 Automatic Fairing

Parametric and geometric continuity of subdivision surfaces will be thoroughly analyzed. For a given subdivision scheme, sufficient and necessary condition in terms of control points will be established for each order of smoothness. *Constrained optimization* technique and *refinement schemes* that will lead to subdivision surfaces with higher order of smoothness within the given error tolerance will be developed.

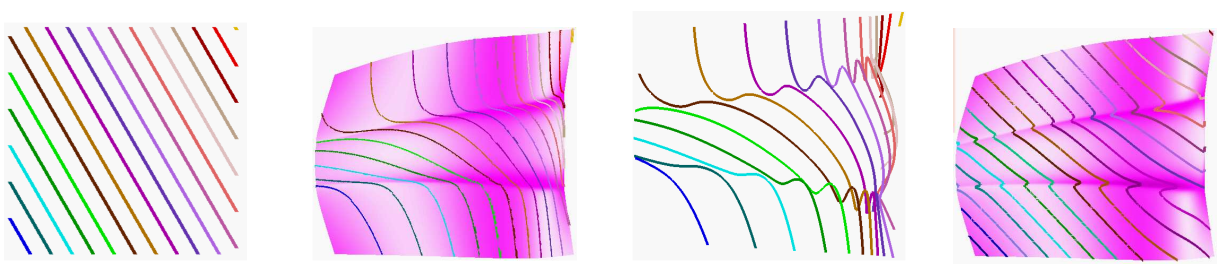
The study will also develop methods for detecting and correcting local shape anomalies. One possible approach is composed of three steps:

1. Create a *fairness indicating model* for the subdivision surface. The model is composed of a family of curves on the subdivision surface.
2. Fair curves of the fairness indicating model. This may include reparametrization of some of the curves.
3. Adjust control points of the subdivision surface to match the faired curves of the fairness indicating model.

The fairness indicating model should be intuitive enough for an average user to understand and efficient enough for an interactive design environment. The *shadowgraph line model* recently develop by us is a possible choice. If $\mathbf{C}(t)$ is a curve in the domain of a surface $\mathbf{S}(u,v)$ and $\mathbf{P}$ is a 3D plane, a *shadowgraph line* is the intersection of the plane $\mathbf{P}$ with the normal of the surface, $\mathbf{N}(u,v)$, at the points of the curve $\mathbf{S}(\mathbf{C}(t))$. $\mathbf{P}$ is called a *projection plane* and $\mathbf{C}(t)$ is called a *source curve*. Given a set of source curves in the domain of a surface $\mathbf{S}(u,v)$ and a projection plane $\mathbf{P}$, a *shadowgraph line model* for $\mathbf{S}(u,v)$ is the family of *shadowgraph lines* created on $\mathbf{P}$ for the given source curves. If the projection plane $\mathbf{P}$ is defined by point $\mathbf{A}$ and vector $\mathbf{Z}$ then $\mathbf{F}$ can be expressed as follows:

$$\mathbf{F} = \mathbf{S}(u,v) + \left[\frac{(\mathbf{A} - \mathbf{S}(u,v)) \cdot \mathbf{Z}}{\mathbf{N}(u,v) \cdot \mathbf{Z}}\right] \mathbf{N}(u,v). \tag{3}$$

An example of SG lines is shown in Figure 5, with (a) being the source curves, (b) being a shaded image with the mapped source curves, (c) being the corresponding shadowgraph line model, and (d) being a shaded image with a highlight line model. The light sources of the highlight line model are in the same orientation as the source curves.



(a) (b) (c) (d)

Figure 5: (a) Source curves, (b) shaded image with mapped source curves, (c) shadowgraph line model, and (d) shaded image with a highlight line model of an automotive hood.

Several points can be made immediately about the shadowgraph line model. First, there is an *analytical representation* for each shadowgraph line (not available for the reflection line model and the highlight line model). Therefore, there is no cost in getting a representation for a shadowgraph line at all. Second, a shadowgraph line is also sensitive to the change of normal directions because it is determined by the normal of the surface as well. Actually, since the surface normal passes through a more complex source curve, instead of a straight line in the light source plane, a shadowgraph line usually magnifies the variation of curvature of a surface more than a highlight line (see Figure 5(c) and (d)) and, consequently, has a better capability in identifying irregularities of a surface. Third, while linear light sources are always used in the highlight line model to ensure the solvability of the parameter finding process, the source curves in the shadowgraph line model can be of any shape. This is an important feature because it allows one to use complex source curves to generate shadowgraph lines which is not possible for the highlight line model and, therefore, it is easier for a viewer to detect irregularities of a surface without rotating the source curves.

An *indexing scheme* will be used to speed up the generation of the shadowgraph line family for this approach. For each node of the approximating mesh, two numbers, I and R, will be computed as follows:

$$D = m_c \times I + R, \qquad 0 \leq R < m_c,$$

where m_c is the distance between two neighboring source curves. Then for each edge, we use the following rules to determine if there are any shadowgraph line nodes contained in this edge:

- If the end points of the edge have the same I value, then either there is no shadowgraph line node between the endpoints of the edge, or the entire edge is part of a shadowgraph line.
- If the end points of the edge have different indicators, I_1 and I_2, then there are $|I_2 - I_1|$ shadowgraph line nodes between its end points. These nodes can be calculated using an incremental method.

We will develop new techniques to automatically identify irregularities of shadowgraph lines. The regions that contain the irregularities will be replaced with curve segments with desired shape. The smoothness of the shadowgraph lines after the replacement process is guaranteed. The control points of the subdivision surface are then adjusted to match the new shadowgraph lines, i.e., control points of the subdivision surface are adjusted so that the new subdivision surface would have the modified shadowgraph lines as its shadowgraph lines. This requires the establishment of formula that describe the relationship between the control points of the subdivision surface and its shadowgraph lines.

Another possible approach that will be considered is composed of two steps:

1. Create a fairness indicating model for the subdivision surface. The model is a function or functions defined on the parameter space of the subdivision surface, but it is not made up of curves. This model should be intuitive to understand. One possibility, for example, is to use different colors to represent different curvature ranges of the surface.
2. Identify and remove anomalies of the subdivision surface based on the above model.

3.4.3 Shape Design

Interpolation-based Shape Design Techniques: The task here is the construction of a control mesh that guarantees the interpolation of a given net of curves by the limit surface of the control mesh for a given subdivision scheme. We will focus on subdivision schemes for quadrilateral meshes (which can be easily converted to triangular meshes). The study consists of three steps:

1. define the topology of the control mesh from the topology of the given net of curves so that the control mesh with such a topology would interpolate the given net of curves with a good shape;
2. establish relationship between the control points of the target subdivision surface and the given curves;
3. develop methods to refine the original mesh of the target subdivision surface so that the given net of curves will be interpolated by the subdivision surface within the given error tolerance.

Manipulating Control Points of Subdivision Surfaces: Manipulation of subdivision surfaces includes manipulating their control points and manipulating points of the subdivision surfaces directly. Our approach for the first case will be different from the traditional methods.

In a traditional model, control points are usually connected by *springs* to ensure that related control points are moved according to physics laws if a control point is moved to a new location by the user. Our approach, however, will be based on *shape optimization*. New *shape indicators* will be created to ensure that related control points are adjusted through an *optimal shape deformation* if a control point is dragged to a new location by the user. This approach is an improvement of the traditional approaches in that it includes the effect of strain energy in the optimization process. The significance of the new approach is that it also considers the impact of the movement of a control point on the smoothness of the surface and tries to reduce the negative impact of such a movement to a minimum level.

Direct Manipulation of Subdivision Surfaces: It is more intuitive for a user to manipulate the subdivision surface directly, especially for those non-interpolating subdivision schemes. Here *direct manipulation of a subdivision surface* refers to the process of dragging points of the subdivision surface directly, instead of dragging its control points. Mathematically speaking, dragging a surface point $\mathbf{V}$ to a new location $\mathbf{V}'$ means changing the control points of the subdivision surface so it would interpolate $\mathbf{V}'$ instead of $\mathbf{V}$. This is more appealing to the user because user is no longer responsible for finding the new locations of the control points; it is the job of the software now. The user is allowed to manipulate only one point at a time. If several points of the subdivision surface have to be manipulated, one has to do it one by one. The study consists of three steps:

1. developing efficient methods to calculate intersection points of a line with the subdivision surface so that the user can use a ray to identify the point to be manipulated. An alternative is to use sampled points, such as points of the "grids", as handles for the user to manipulate the surface. The reason that the word "grids" is quoted is to emphasize the fact that the parameter space of a subdivision surface is not necessarily to be rectangular. The first approach is more natural and convenient for a user to use, but the alternative approach is easier to implement.

2. developing formula that represents the relationship between displacement of the control points and displacement of a point of the subdivision surface so that by putting the displacement of a subdivision surface point into the formula, one can easily find required displacement of the control points (for the new surface to pass through the new location).

3. adopting the idea of the above approach ("manipulating control points of the subdivision surface") in this approach so that through an *shape optimization* process, the surface would not only pass through the new location of the moved point, but also has the best possible shape.

3.4.4 Surface Trimming

Calculating Trimming Curves of Subdivision Surfaces: This work is the core of all Boolean operations. To allow a user to create trimmed subdivision surfaces of his own choice, one needs the capability of computing intersection curves (trimming curves) of two subdivision surfaces, or a subdivision surface and a parametric surface. An intuitive approach is to follow the traditional *marching* technique [5][55][58] for parametric surfaces with the help of our error estimating technique for subdivision surfaces. However, our initial experience shows that in this case the number of vertices required for the trimming curve increases exponentially with respect to the precision requirement. Hence, the traditional marching technique is not a good choice for trimming operation of subdivision surfaces. We will develop new techniques to find and represent trimming curves of subdivision surfaces. The study includes:

- developing new representation scheme for trimming curves. The new scheme would take much smaller data size to represent a trimming curve (for a given error tolerance), and it would facilitate the rendering process and satisfy the LOD (level of details) requirement. Besides, the new representation scheme should satisfy certain requirements for the mesh generation process because trimming curves play an important role in mesh generation of trimmed surfaces as well.

- developing new trimming curve computing methods. The new methods would ensure that features of the new representation scheme are considered in the trimming curve computation process and would emphasize more on precision and robustness than on efficiency. The *divide-and-conquer* and the *refinement* techniques will be considered for the new methods. Our initial experience with these techniques show that they are promising both in efficiency and accuracy (within any arbitrarily small error tolerance).

Mesh Generation for Trimmed Subdivision Surfaces: We will develop new methods to generate approximating meshes for trimmed subdivision surfaces for applications in finite element analysis (FEM) and surface rendering. These two application areas have different requirements on the input meshes. Therefore, different mesh generation techniques will be developed for these applications. For FEM application, the study consists of two steps:

1. developing new methods to remesh the original control mesh or an intermediate mesh (half product) so the resulting mesh is composed of quadrilateral or triangular elements with reasonably good shape only.

2. developing new subdivision methods for the remeshed (refined) meshes so that the resulting mesh approximates the same limit surfaces with an error that can be made arbitrarily small. The reason for doing so is because the remeshing process could change the nodes and topology of the original mesh. Therefore, to hold the resulting limit surface unchanged, new subdivision schemes have to be developed and used.

For surface rendering application, the study includes

- developing new, adaptive mesh generation methods for trimmed subdivision surfaces. For rendering application, shape of the mesh faces is not that important; efficiency and robustness are more important issues. Adaptive methods are used here because they can speed up the mesh generation process and reduce the number of nodes in the resulting mesh.
- developing new methods to generate approximating mesh for a surface composed of several trimmed subdivision surfaces or a solid bounded by several trimmed subdivision surfaces. In this case, we need to take care the crack problem between or among different trimmed surfaces (for non-manifold case). Cracks may occur due to different error tolerances or different subdivision schemes on different surfaces. Note that by applying different subdivision schemes on adjacent surfaces, one could produce different nodes on the boundary curves shared by those surfaces.

3.5 Work Plan and Time Table

Three years are planned for the project beginning with July of 2004. In the first year the following activities are scheduled:

- September 1, 2004 → December 31, 2004
  - develop basic parallel label-driven tessellation techniques
  - develop numerical formula to precisely estimate the error between control mesh and limit surface
  - develop remeshing technique subject to FEM requirements
  - develop subdivision schemes for the remeshing process that guarantee approximation of the same limit surface with arbitrarily small error
  Milestones: tools for subdivision surface tessellation
  tools for FEM mesh generation for subdivision surfaces
  a mesh error evaluator
- January 1, 2005 → April 30, 2005
  - develop techniques to compute subdivision level for given error tolerance
  - study the relationship between the control points and the bounds of derivatives for subdivision surfaces
  - develop new label assignment technique to speed up the label-driven tessellation process
  - develop basic parallel error-driven tessellation techniques
  Milestones: new mesh generation tools for subdivision surfaces
  a report on derivative bound calculation
  a subdivision level evaluator
- May 1, 2005 → August 31, 2005
  - develop constrained optimization techniques and refinement schemes to obtain subdivision surfaces with higher order of smoothness within the given error tolerance I
  - create fairness indicating model for subdivision surfaces
  - develop fairing techniques for curves in the fairness indicating model I

- develop fairness indicating model based anomaly detecting techniques for subdivision surfaces
  Milestones: tools for smoothing subdivision surfaces
  tools for fairing smoothness indicating curves
  a fairness indicating model for subdivision surfaces
  a report on anomaly detection

The activities for the second year are scheduled as follows:

- September 1, 2005 → December 31, 2005
  - develop new label assignment technique to speed up the error-driven tessellation process
  - develop constrained optimization techniques and refinement schemes to obtain subdivision surfaces with higher order of smoothness within the given error tolerance II
  - develop optimal rendering techniques for the fairness indicating model
  Milestones: a report on getting subdivision surfaces with higher smoothness order
  a report on fairness indicating model rendering
  a report on optimal tessellation techniques
- January 1, 2006 → April 30, 2006
  - develop automatic subdivision surface fairing techniques by adjusting control points to match faired curves in the fairness indicating model
  - develop fairing technique for curves in the fairness indicating model II
  - develop anomaly detecting techniques for subdivision surfaces by checking on the control point layout
  - develop constrained optimization techniques for removing anomaly of subdivision surfaces by re-arranging control points
  Milestones: tools for subdivision surface fairing
  a report on fairing smoothness indicating curves
  a report on anomaly detection and removal
- May 1, 2006 → August 31, 2006
  - develop techniques to obtain topology of the control mesh from topology of the given net of curves
  - develop methods to determine desired locations of the control points so the targeted subdivision surface would be close to the given net of curves
  - develop adjustment and refinement skills to make the targeted subdivision surface interpolate the given curves within a given error tolerance
  Milestones: a report on control mesh topology construction
  a report on control mesh position calculation
  a report on control mesh refinement
  tools for performing subdivision surface interpolation of given net of curves

The activities scheduled for the third year are:

- September 1, 2006 → December 31, 2006
  - develop shape optimization techniques for direct subdivision surface control point manipulation
  - develop efficient methods to calculate intersection points of a line and a subdivision surface
  - develop formula to estimate required control point displacement in order to get desired subdivision surface displacement
  - develop shape optimization method with step size control for subdivision surfaces
  Milestones: tools for subdivision surface shape optimization
  a report on line-subdivision-surface intersection
  a report on relationship between control point displacement and subdivision surface displacement
- January 1, 2007 → April 30, 2007
  - develop intersection curve computation techniques for subdivision surfaces (between two subdivision surfaces or between a subdivision surface and a parametric surface)
  - develop trimming curve reparametrization techniques

- develop adaptive mesh generation methods for trimmed subdivision surfaces
  Milestones: tools for performing subdivision surface intersection
  tools for mesh generation on trimmed subdivision surfaces
  a report on trimming curve computation for subdivision surfaces

- May 1, 2007 $\rightarrow$ August 31, 2007
  - develop new methods to generate approximating mesh for the boundary of a solid composed of trimmed subdivision surfaces
  - develop UNION, INTERSECTION and DIFFERENCE operation techniques for subdivision surfaces
  Milestones: tools for performing Boolean operations on subdivision surfaces
  a report on mesh generation for surfaces combined of trimmed subdivision surfaces
  several reports on Boolean operations

3.6 Impact (Significance) of the Proposed Research

Advancing discovery and understanding: Results of the proposed research will provide significant advancement in subdivision surface based modeling. Results in the first area will not only provide us with a precise and yet efficient way to generate input to the FEM and surface rendering processes, but also a better way to understand the spatial properties of subdivision surfaces such as convexity and curvature distribution and, hence, provide us with an opportunity to address the manufacturability issues in the surface development stage. Results in the second area will significantly shorten the subdivision surface fairing process and make CAD more proactive in the product development process for subdivision surfaces. Results in the third area will provide design engineers with effective tools to sculpt a subdivision surface into desired shape and direct manipulation of surface curvature and variation of curvature that has never been done before and, therefore, will increase the productivity of a design engineer dramatically. Results in the fourth area will provide us with the needed core technologies in performing Boolean operations to form more complicated objects and, consequently, will make subdivision surface based modeling operations more reliable. Overall, results of the proposed research will provide the design industry with a solution set for most subdivision surface based modeling operations and, consequently, provide support to make subdivision surfaces the next generation surface representation for CAD/CAM applications.

Promoting teaching, training, and learning: The proposed project will provide graduate students at the University of Kentucky an excellent experience to work on subdivision surface based modeling techniques. This unique experience will make them invaluable researchers or engineers for the CAD/CAM industries of the United States. The results of the proposed research will also enrich curriculum development at the University of Kentucky, particularly in the areas of geometric modeling and CAD/CAM. The PI of this proposal has already integrated his research results into two graduate courses, *Computer Aided Geometric Design* (CS631) and *Freeform Solid Modeling* (CS630) (some subdivision surface representation and subdivision surface based modeling results have already been included in both courses). The results of the proposed research will be integrated into these courses as four additional topics on subdivision surface modeling.

Increasing participation of minority groups: There is already a female member (Alice J. Lin) in the PhD student group who would participate in the proposed research work. We will also recruit minority students to participate in the proposed project through the department's honor program.

Enhancing broad dissemination: The PI has established collaboration with several companies such as SDRC (currently called EDS PLM-Solution), Ford, and Honda through funded projects from these companies (three-year research grant from Ford and two one-year research grants from Honda) or required technology transfer (to SDRC, required by Ford). The CAD modeling group at the Ford Research Center and the Curves and Surfaces Group at SDRC have already expressed their interest in reviewing the results of this project. Therefore, in addition to publication, technology sharing will also be a part of the PI's research agenda.

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4 Biographical Sketches

Fuhua (Frank) Cheng (PI)

Education:

8/80-5/82	Ph.D., Applied Math. & Computer Science, Ohio State University
8/78-5/80	M.Sc., Computer Science, Ohio State University
6/77-5/78	M.Sc., Mathematics, Ohio State University
9/69-6/75	M.Sc. Mathematics, Tsinghua University, Taiwan, R.O.C.

Employment:

7/98-	Prof., Dept. of Computer Science, Univ. of Kentucky
9/93-8/94	Project Director, Olympus Optical Co., Tokyo, Japan
9/93-3/94	Visiting Researcher, Dept. of Information Science, Univ. of Tokyo, Japan
9/93-8/94	Visiting Prof., Shape Modeling Lab., Univ. of Aizu, Japan
7/89-6/98	Assoc. Prof., Dept. of Computer Science, Univ. of Kentucky
8/86-6/89	Assist. Prof., Dept. of Computer Science, Univ. of Kentucky
8/82-7/86	Assoc. Prof., Institute of Computer Science, Tsinghua Univ., Taiwan, R.O.C.

Research Interest:

Geometric/solid modeling, computer graphics, collaborative CAD, parallel computation in graphics and geometric modeling, approximation theory

Synergistic Activities:

1. Program Committee: CAD'06, *Geometric Modeling and Processing 2006*, *24th Computer Graphics International Conference (CGI2006)*, ISICS2006, *IASTED Int. Conf. Computer Graphics and Imaging (CGIM 2007)*, *IASTED Int. Conf. Graphics and Visualization in Engineering (GVE 2007)*, CAD'07, *11th ACM Symp. Solid and Physical Modeling (2007)*.

2. Editing: *J. Information and Computational Science*, *Computer Aided Design & Applications*, *J. Math Analysis & Approximation Theory*, *J. Computer Aided Design & Computer Graphics*

3. Worked with Mechanical Engineering of the University of Kentucky and researchers from three other universities (University of Missouri - Rolla, University of Rhode Island, and University of Louisville) on an NSF ERC proposal (not selected for the second stage) and working with Mechanical Engineering and Biomedical Engineering of the University of Kentucky on an *Interactive Graduate Education and Research Traineeship (IGERT)* proposal.

4. Courses being taught at the University of Kentucky in the CAD area are (1) Free-form Solid Modeling (CS630) and (2) Computer-Aided Geometric Design (CS631).

5. Work ongoing in research lab on *Subdivision Surface based One-Piece Representation*, *Tessellation, Fairing, Shape Design and Trimming Techniques for Subdivision Surface based Modeling*, *On New Algorithms of Curve and Surface Modeling Based on Probabilistic Type Operators and Probability Distribution*

5 Publications Most Closely Related to Proposed Research:

1. Similarity based Interpolation using Catull-Clark Subdivision Surfaces (with S. Lai), to appear in *The Visual Computer*.

2. Parametrization of Catull-Clark Subdivision Surfaces and Its Applications (with S. Lai), *Computer Aided Design & Applications*, 3,1-4 (2006), 513-522.

3. Subdivision Depth Computation for Catmull-Clark Subdivision Surfaces (with Junhai Yong), *Computer Aided Design & Applications*, 3,1-4 (2006), 485-494.

4. Adaptive Subdivision of Catmull-Clark Subdivision Surfaces (with J. Yong), *Computer-Aided Design & Applications* 2,1-4 (2005), 253-261.
5. Constrained Scaling of Catmull-Clark Subdivision Surfaces (with S. Lai and S. Zou), *Computer-Aided Design & Applications* 1,1-4 (2004), CAD04, 7-16.

5 Selected Other Publications:

1. Matrix based Subdivision Depth Computation for Extra-Ordinary Catmull-Clark Subdivision Surface Patches (with Gang Chen), Geometric Modeling and Processing - GMP2006, *Lecture Notes in Computer Science*, Vol. 4077, Springer,
2. Voxelization of Catmull-Clark Subdivision Surfaces (with S. Lai), Geometric Modeling and Processing - GMP2006, *Lecture Notes in Computer Science*, Vol. 4077, Springer, 595-601.
3. Near-Optimum Adaptive Tessellation of General Catmull-Clark Subdivision Surfaces (with S. Lai), Advances in Computer Graphics (CGI2006), *Lecture Notes in Computer Science*, Vol. 4035, Springer, 2006, 562-569.
4. Subdivision Depth Computation for Extra-Ordinary Catmull-Clark Subdivision Surface Patches (with Gang Chen and Junhai Yong), Advances in Computer Graphics (CGI2006), *Lecture Notes in Computer Science* Vol. 4035, Springer, 2006, 404-416.
5. Texture Mapping on Surfaces of Arbitrary Topology using Norm Preserving Based Optimization (with S. Lai), *The Visual Computer* 21,8-10 (2005), 783-790.
6. Adaptive Rendering of Catmull-Clark Subdivision Surfaces (with S. Lai), *Proc. 9th Int. Conf. on Computer Aided Design and Computer Graphics (CAD/CG 2005)*, Dec 7-10, 2005, Hong Kong, 125-130.

U.S. Patents:

- Four U.S. Patents pending.

Persons collaborated within last 48 months:

Brian Barsky, Yifan Chen, Shuhua Lai, Kenjiro T. Miura, Minetada Osano, J. Sone, Paul Stewarts, William Toll, Minoru Ueda, Gang Chen, Junhai Yong, Xiaoming Zeng, Caiming Zhang, Pifu Zhang

Graduate Advisor:

Prof. Ranko Bojanic - Ohio State University

Graduate Student and Postdoctoral Advisees (past 60 months):

1. Dr. Shuhua Lai - Virginia State University, Beijing, China
2. Dr. Junhai Yong - Tsinghua University, Beijing, China
3. Dr. Xuefu Wang - Google, CA
4. Dr. Yong Zhou - UCLA
5. Dr. Huaijun Wu - MIT
6. Dr. Caiming Zhang - Shandong University, Jinan, China
7. Dr. Pifu Zhang - Dalhousie University, Halifax, Canada
8. Dr. William Toll - Taylor University, Indiana