Subdivision Surface based One-Piece Representation

Shuhua Lai

Department of Computer Science,
University of Kentucky

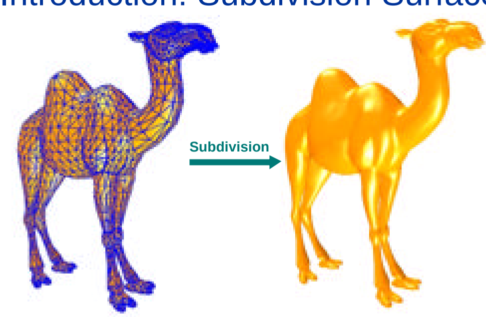
Outline

1. Introduction
2. Parametrization of General CCSSs
3. One-Piece through Interpolation
4. One-Piece through Boolean Operations
5. Simplification by Adaptive Tessellation
6. Simplification by Multiresolution Analysis
7. Conclusion & Future Work

Abbreviation:

CCSS = Catmull-Clark Subdivision Surface

Introduction: Subdivision Surface



Subdivision surfaces have already been used in

- Pixar's Renderman
- Alias|Wavefront's Maya
- Nichimen's Mirai
- Newtek's Lightwave 3D

⋮

What is so special?

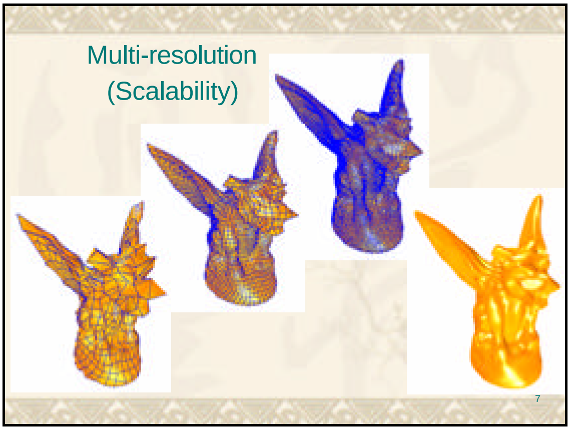
Multi-resolution (Scalability)

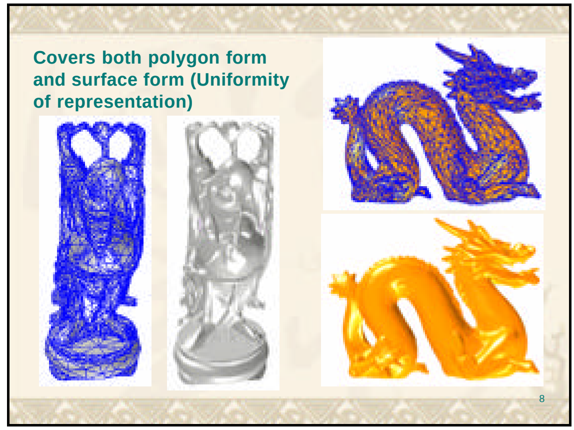
One piece representation™ (arbitrary topology)

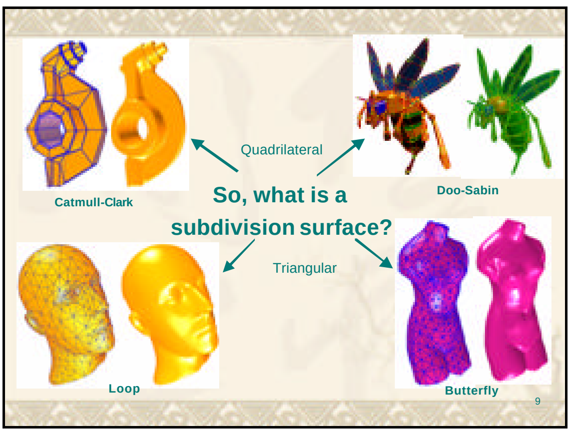
Numerical stability

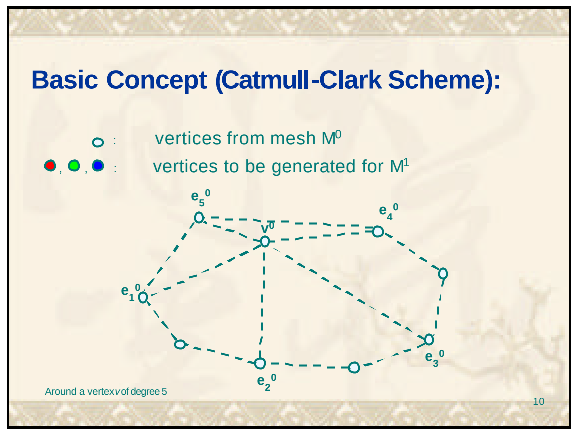
Covers both polygon form and surface form (Uniformity)

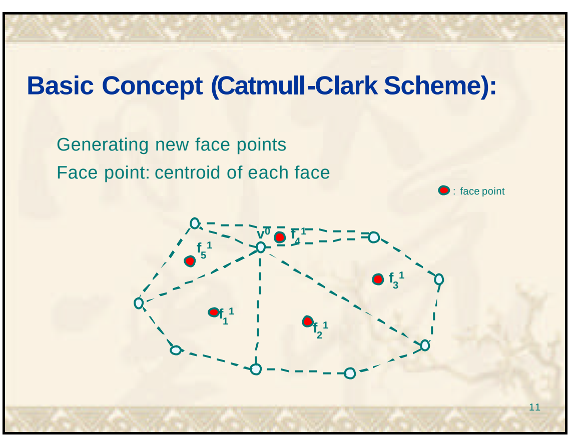
Code Simplicity

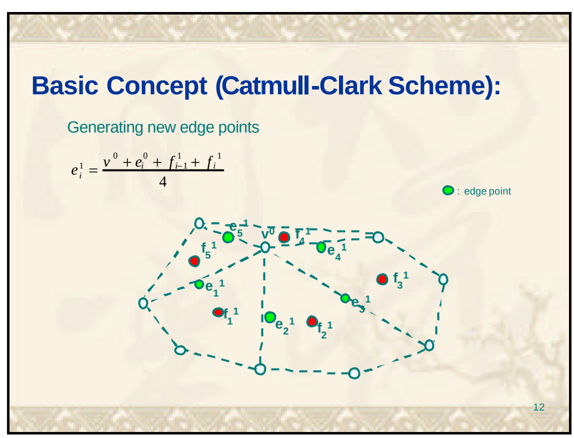

Multi-resolution
(Scalability)
7


Covers both polygon form and surface form (Uniformity of representation)
8


Catmull-Clark
Quadrilateral
Doo-Sabin
So, what is a subdivision surface?
Triangular
Loop
Butterfly
9


Basic Concept (Catmull-Clark Scheme):
: vertices from mesh M^0
, , : vertices to be generated for M^1
e_5^0
v^0
e_4^0
e_1^0
e_3^0
e_2^0
Around a vertex v of degree 5
10

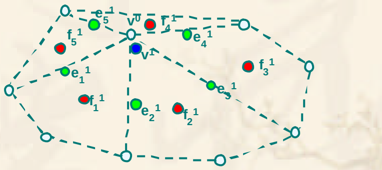

Basic Concept (Catmull-Clark Scheme):
Generating new face points
Face point: centroid of each face
: face point
v^0
f_4^1
f_5^1
f_3^1
f_1^1
f_2^1
11


Basic Concept (Catmull-Clark Scheme):
Generating new edge points
$e_i^1 = \frac{v^0 + e_i^0 + f_{i-1}^1 + f_i^1}{4}$
: edge point
e_5^1
v^0
f_4^1
e_4^1
f_5^1
f_3^1
e_1^1
e_3^1
f_1^1
e_2^1
f_2^1
12

Basic Concept (Catmull-Clark Scheme):

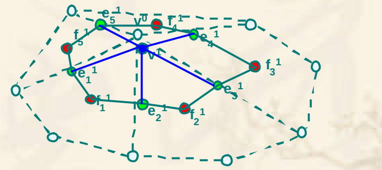
Generating new vertex points

$$v^1 = \frac{n-2}{n} v^0 + \frac{1}{n^2}\sum e_i^0 + \frac{1}{n^2}\sum f_i^1$$



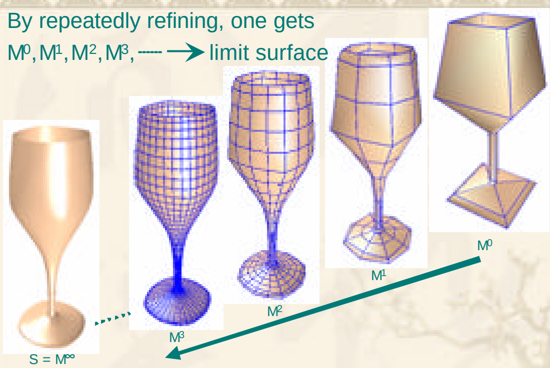
Basic Concept (Catmull-Clark Scheme):

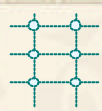
Forming new edges



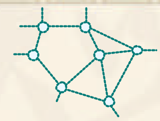
By repeatedly refining, one gets

$M^0, M^1, M^2, M^3, \cdots \rightarrow$ limit surface





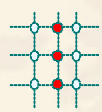
NURBS



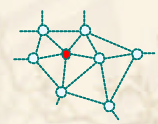
Catmull-Clark

Modeling made much easier. Why?

- No restrictions on the topology of the control points
- Local refinement is possible

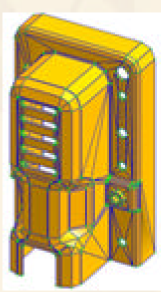


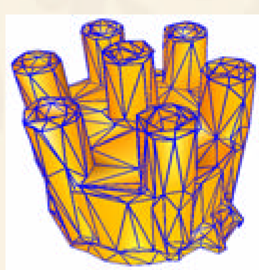
NURBS



Catmull-Clark

Examples of control mesh



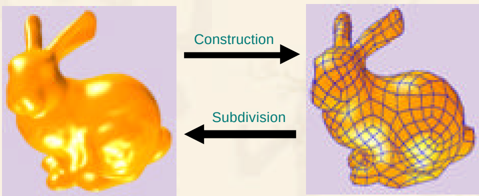


Motivation

- Although subdivision surfaces are capable of modeling complex shape of arbitrary topology, methods on how to build the control mesh of a complex surface are not presented yet.

Task

To construct a sparse mesh structure for any given model such that The CCSS of the constructed mesh is the given model.



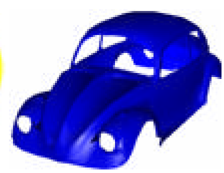
Objectives

1. Develop necessary mathematical theories and geometric algorithms to support subdivision surface based one-piece representation
2. Build a system that supports subdivision surface based one-piece representation.

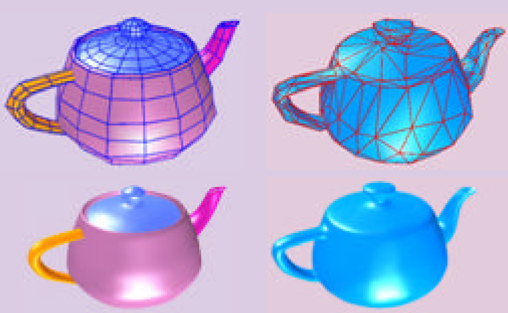
Subdivision surface based one-piece representation

- Represent any model with only one subdivision surface no matter how complicated the object's topology or shape. No decomposition of the object into simpler components is necessary. Hence the number of parts in the final representation is always the minimum: one.

Examples of One piece representation



Multi- V.S. One-Piece



Is One Piece Representation Good?

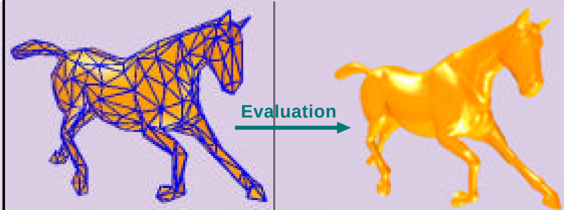
- **Management:** **Simpler**
- **Rendering:** **More efficient**
- **Machining:** **More accurate**
- **Animation:** **Crack free**

Contributions of the dissertation

- **Parametrization** of CCSSs
- **Interpolation** of meshes of arbitrary topology
- **Voxelization** of free-form solids
- **Boolean operations** on free-form solids
- **Adaptive tessellation** of CCSSs
- A **system** that supports subdivision surface based one-piece representation is built
- **Others**

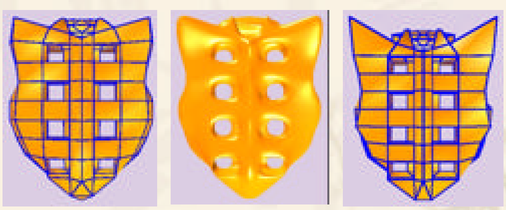
Contributions (cont.)

- Parametrization and Evaluation of CCSSs



Contributions (cont.)

- Interpolation of meshes of arbitrary topology



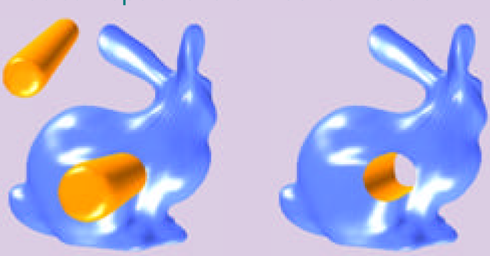
Contributions (cont.)

- Voxelization of free-form solids

Given Model 128 X 128 X 128 512 X 512 X 512

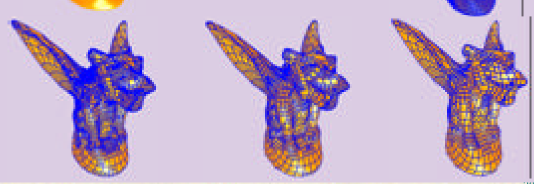
Contributions (cont.)

- Boolean operations on free-form solids



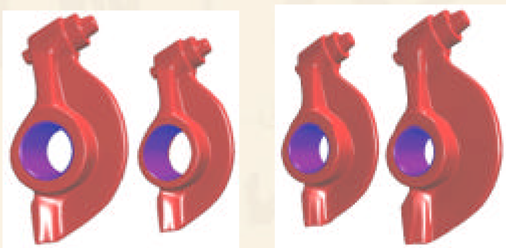
Contributions (cont.)

Adaptive tessellation



Contributions (cont.)

- Constrained Scaling of CCSSs



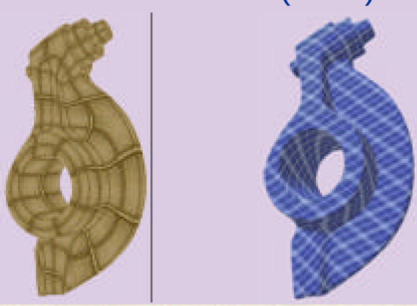
31

Contributions (cont.)

- Texture Mapping on Meshes of Arbitrary Topology

32

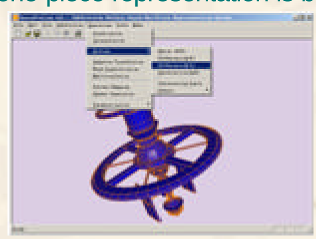
Contributions (cont.)



33

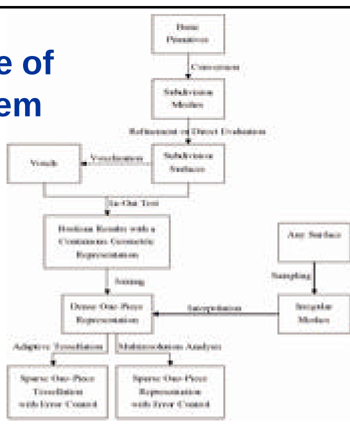
Contributions (cont.)

- A **system** that supports subdivision surface based one-piece representation is built



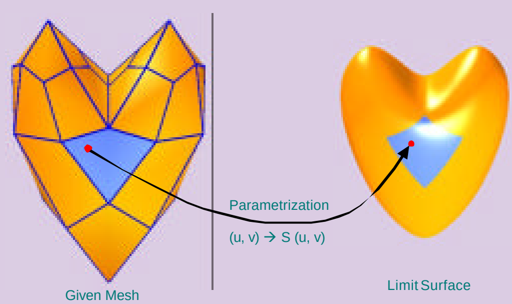
34

Structure of the system



35

Parametrization of General CCSSs

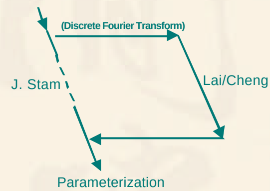


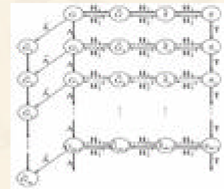
36

Work on Subdivision Surface Parameterization

1. J. Stam (1998)
2. D. Zorin, *et al*. (2002)
3. S. Lai, F. Cheng (2005)

The Extended Subdivision Diagram





Explicit Parametrization of General CCSSs

$$S(u,v) = W^T K^m \sum_{j=0}^{n+5} l_j^{m-1} M_{b,j} G$$

Explicit Parametrization (Cont.)

- $W = [1, u, v, u^2, uv, v^2, u^3, u^2v, uv^2, v^3, u^3v, u^2v^2, uv^3, u^3v^2, u^2v^3, u^3v^3]$;
- K = Diag(1,2,2,4,4,4,8,8,8,8,16,16,16,32,32,64);
- λ_j = eigenvalue of the subdivision matrix;
- $M_{b,j}$ = constant matrix depending on b and j
- $G = [V, E_1, E_2, \ldots, E_n, F_1, \ldots, F_n, I_1, I_2, \ldots, I_7]$;

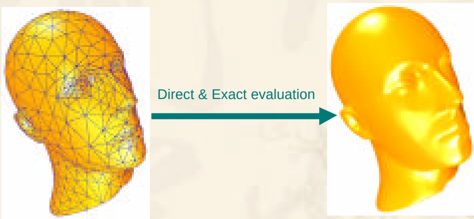
Properties around extraordinary points

$$
\begin{aligned}
S(0,0) &= [1,0,0,0,0,0,\cdots,0] \bullet M_{2,n+1} \bullet G \\
\frac{\partial S(0,0)}{\partial u} &= [0,1,0,0,0,0,\cdots,0] \bullet M_{2,2} \bullet G \\
\frac{\partial S(0,0)}{\partial v} &= [0,0,1,0,0,0,\cdots,0] \bullet M_{2,2} \bullet G \\
\frac{\partial^2 S(0,0)}{\partial u \partial u} &= [0,0,0,1,0,0,\cdots,0] \bullet M_{2,2} \bullet G \\
\frac{\partial^2 S(0,0)}{\partial u \partial v} &= [0,0,0,0,1,0,\cdots,0] \bullet M_{2,2} \bullet G \\
\frac{\partial^2 S(0,0)}{\partial v \partial v} &= [0,0,0,0,0,1,\cdots,0] \bullet M_{2,2} \bullet G
\end{aligned}
$$

Applications of the new parameterization technique

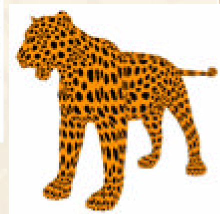
- Surface Evaluation
- Texture Mapping
- Boolean Operations
- Surface Trimming
- Adaptive Tessellation
- Animation

Surface Evaluation



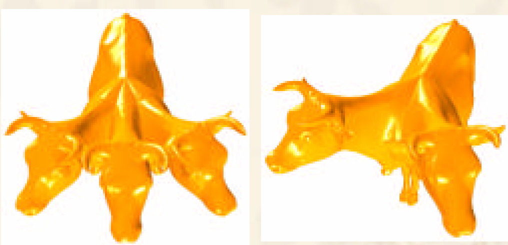
43

Texture Mapping


44

Boolean Operations

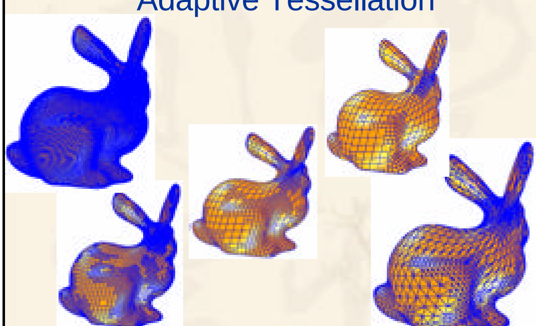


45

Surface Trimming

46

Adaptive Tessellation



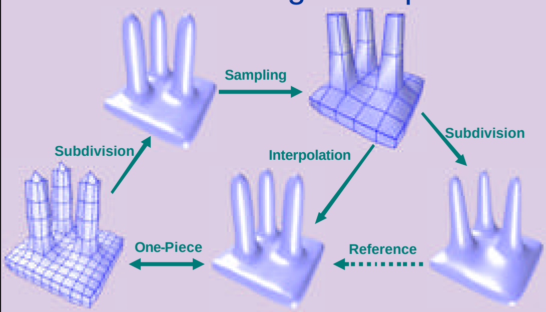
47

Two Approaches for One-Piece Representation

1. By Interpolation
2. By Boolean Operations

48

One-Piece through Interpolation



49

Similarity Based Interpolation

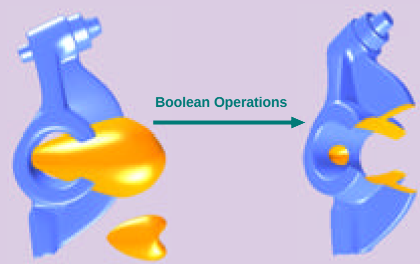
- Result in a one-piece represented mesh.
- Assume the interpolating surface should be similar to the limit surface of the given mesh
- Surface fairing is not needed.
- No system solvability problem.
- Can handle both open and closed meshes.

50

Interpolating Open Meshes

51

One-Piece by Boolean Operations



52

Voxelization: Basic Idea

Perform adaptive (midpoint) subdivision in parameter space until each subpatch is small enough so that it can be voxelized using only its four corners

53

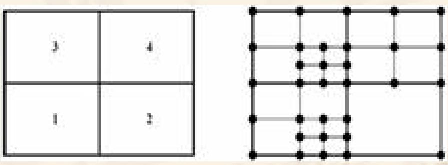
Related Work

Two major voxelization techniques:

- 3D Scan-line Algorithms [6,7,8,9]
  - Only good for polygonal meshes
- 3D Spatial Enumeration Algorithms[10]
  - Computationally expensive.

54

Recursive 2D Parameter Space Subdivision



Separability, Accuracy and Minimality

$\forall P \in S, \exists Q \in \overline{S},$ such that $P \in Q$

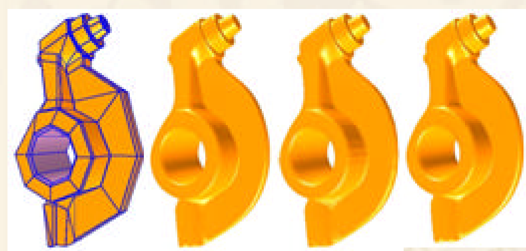
$\forall Q \in \overline{S}, \exists P \in S,$ such that $P \in Q$

Note that here P is a 3D point and Q is a voxel, which is a unit cube. S is the CCSS and $\overline{S}$ is the voxelization of S.

Applications of Voxelization

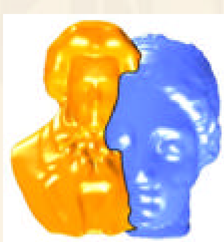
- Visualization of Complex Scenes
- Integral Properties Measurement
- Intersection Curves Determination
- Performing CSG Operations
- Performing Boolean Operations

Rendering of Voxels

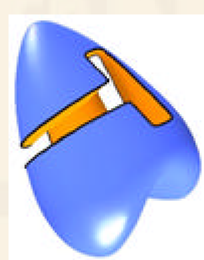


Mesh | Limit Surface | Point based Rendering | Splat based Rendering

Intersection Curves



Boolean Operations

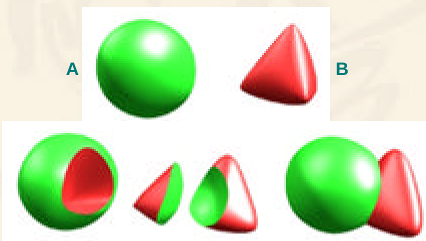


CSG Operations

Boolean Ops Based on Voxelization

- **Inside Voxels**
- **Boundary Voxels**
- **Outside Voxels**

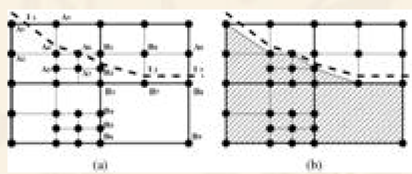
Approach is Similar for all kinds of Boolean Ops



A-B A*B B-A A+B

Figures from [7]

Boolean Ops Based on Recursive 2D Parameter Space Subdivision



Local Voxelziation

- Impossible for 3D voxelization methods.
- Easy to be implemented in voxelization methods based on recursive 2D parameter space subdivision.
- Can obtain high precision of Boolean operation results

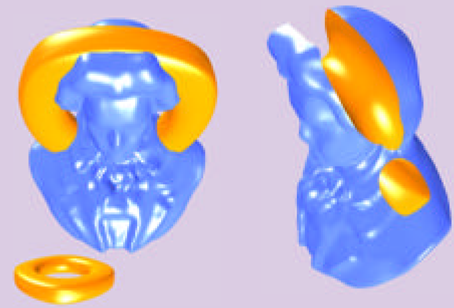
Error Control

$$\begin{cases} \sqrt{d(\bar{u}, \bar{v})} + \sqrt{d(\hat{u}, \hat{v})} & \leq \epsilon/2 \\ \text{size of each voxel} & \leq \epsilon \end{cases}$$

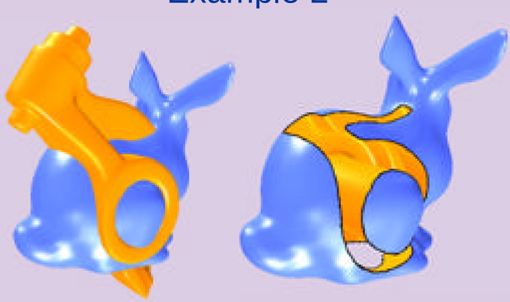
Advantages

- Robust
- Error controllable
- Resulting solids have continuous geometric representations, i.e. one-piece representations.
- Crack free
- Applicable to any sbudivision scheme

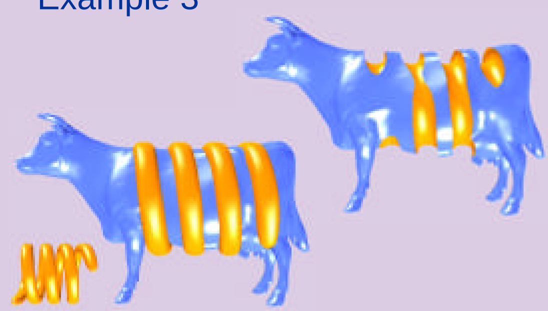
Example 1



Example 2



Example 3



Two Approaches for Mesh Simplification

1. **By Adaptive Tessellation**
2. **By Multi-resolution Analysis**

A disadvantage of CCSSs

- Number of faces in the uniformly refined meshes increases exponentially with respect to subdivision depth

Remedy

- **Adaptive tessellation reduces the number of faces needed to yield a smooth approximation to the limit surface**

Basic Idea

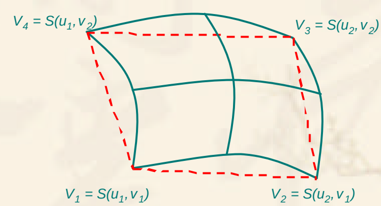
- A flat surface patch should not be tessellated as densely as a surface patch with big curvature.
- The adaptive tessellation process of a surface patch should be performed based on the flatness of the patch.

Objective of Adaptive Tessellation

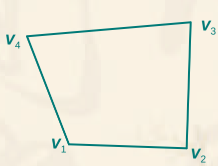
1. Keep the face number of the control mesh small;
2. The approximating model is as precise as possible;
3. Tessellation result is crack free.

Flatness Test

For a patch $S(u,v)$ defined on $u_1 < u < u_2$ and $V_1 < v < v_2$, we try to approximate it with the quadrilateral formed by its four end vertices



Bilinear Parameterization of a quadrilateral



$$L(u,v) = \frac{v_2 - v}{v_2 - v_1}\left(\frac{u_2 - u}{u_2 - u_1}V_1 + \frac{u - u_1}{u_2 - u_1}V_2\right) + \frac{v - v_1}{v_2 - v_1}\left(\frac{u_2 - u}{u_2 - u_1}V_4 + \frac{u - u_1}{u_2 - u_1}V_3\right)$$

Definition of Distance

The distance between the patch (or subpatch) and the corresponding quadrilateral is:

$$D = \max\{ \sqrt{d(u,v)} \mid (u,v) \in [u_1, u_2] \times [v_1, v_2] \}$$

Where

$$d(u,v) = \| L(u,v) - S(u,v) \|^2$$

Finding Extrema

By Mean Value Theorem, the minima and maxima must satisfy at least one of the following 3 conditions:

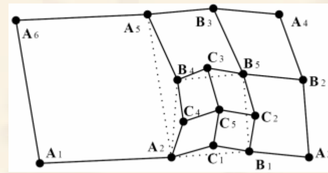
$$\begin{cases} \frac{\partial d(u,v)}{\partial u} = 0 \\ v = v_1 \text{ or } v = v_2 \\ u_1 \le u \le u_2 \end{cases}$$

$$\begin{cases} \frac{\partial d(u,v)}{\partial v} = 0 \\ u = u_1 \text{ or } u = u_2 \\ v_1 \le v \le v_2 \end{cases}$$

$$\begin{cases} \frac{\partial d(u,v)}{\partial u} = 0 \\ \frac{\partial d(u,v)}{\partial v} = 0 \\ (u,v) \in (u_1,u_2) \times (v_1,v_2) \end{cases} \quad (*)$$

Crack Problem

Cracks can be eliminated simply by replacing each boundary of a patch with the one that contains all the evaluated points for that boundary.

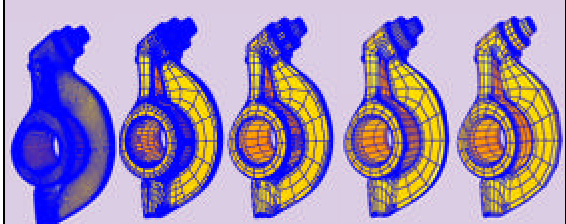


Crack problem

- No crack-detection or crack-elimination is needed

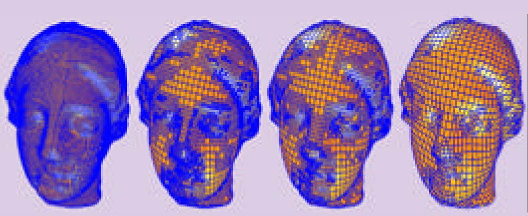
Therefore, no mesh element splitting to eliminate cracks is necessary

Examples



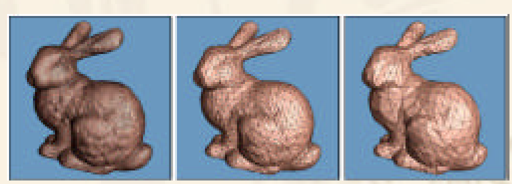
RF = 3% RF = 1% RF = 2% RF = 3% RF = 5%

Examples



RF = 8% RF = 3% RF = 4% RF = 8%

Mesh Simplification by Multi-resolution Analysis



Figures from [111]

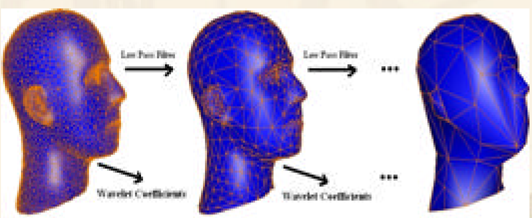
Multi-resolution Analysis

1. There are many methods proposed in the literature;
2. The most well-known one is based on the wavelet transform;
3. Has many applications in computer graphics and computer aided modeling

Applications of Multi-resolution Analysis

- Compression of 3D models
- Simplification of 3D models
- Progressive Display/Transmission
- Level-Of-Detail Control
- Multi-Resolution Editing

Basic Idea of Multi-Resolution Analysis



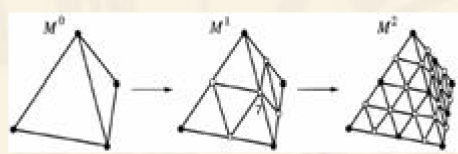
Requirements of Multi-Resolution Mesh Analysis

- The low-resolution versions are good approximations of the original object ;
- The magnitude of a wavelet coefficient reflects its importance;
- Analysis and synthesis should have linear time complexity.

Lounsbery et al's Method

- Based on subdivision surfaces, presents a new class of wavelets that can be applied to functions defined on surfaces of arbitrary topology;
- It can be used for mesh simplification, editing, compression and LOD control etc;
- Only can be applied to surfaces with subdivision connectivity.

Subdivision Connectivity Requirement



the input mesh must have the form of a mesh M_j that results from subdividing a simple mesh M_0 j times.

Figures from [110]

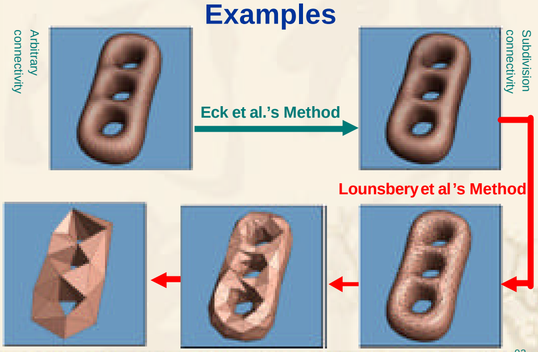
Eck et al.'s Method

- Unfortunately, meshes in practice typically do not meet this requirement;
- Our one-piece representation meshes do not meet this requirement either;
- Eck et al.'s Method overcomes the subdivision connectivity restriction, meaning that completely arbitrary meshes can be converted to multiresolution form.

Two Steps

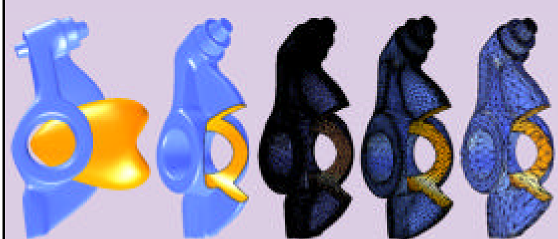
1. First use the method of Eck et al. to convert any mesh M to a mesh M_J which satisfies the subdivision connectivity requirement,
2. Then use the method of Lounsbery et al. to convert M_J to a multiresolution mesh representation.

Examples



Figures from [111]

Examples



Conclusion

- **Mathematical foundation** for subdivision surface based one-piece representation is developed.
- **Two approaches** for constructing subdivision surface based one-piece representation are proposed
- **Two methods** for simplifying control meshes are developed
- A subdivision surface based one-piece representation **system** is built

Future Work

- More convenient & accurate representations for topologically complex 3D objects
- Algorithms for construction, rendering, manipulation, processing, transmission & storage of topologically complex 3D objects
- Interdisciplinary research & applications

Acknowledgement

- Dr. Fuhua (Frank) Cheng
- Dr. Brent Seales
- Dr. Qiang Ye
- Dr. Jun Zhang
- Dr. Grzegorz Wasilkowski
- Dr. Adam Branscum

Acknowledgement

The research work with this dissertation is supported in part by U.S. National Science Foundation (NSF) under grants

- DMI-9912069;
- DMS-0310645;
- DMI-0422126

Acknowledgement

Some data files are downloaded from:

- The Stanford 3D Scanning Repository, http://graphics.stanford.edu/data/3Dscanrep/;
- Dr. Hugues Hoppe: http://research.microsoft.com/~hoppe;
- Dr. Peter Schroder: http://www.cs.caltech.edu/~ps/;
- Dr. Denis Zorin: http://mrl.nyu.edu/~dzorin;
- Dr. Michael Garland: http://graphics.cs.uiuc.edu/~garland.

Thank you !

Any Questions?