Computer-Aided Design and Realization of Geometrical Sculptures

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ABSTRACT

The use of computer-aided design tools in the conception and realization of large-scale geometrical bronze sculptures is described. An inspirational piece of sculpture is analyzed and then captured in procedural form including several design parameters. These parameters not only allow the sculpture to be scaled to different sizes and individually optimized for each scale, but also allow one to design new sculptures that lie in the same conceptual family. The parameterized representation takes care of constraints and limitations in several of the implementation steps and provides additional aids for the assembly of a large sculpture from many smaller and more easily manufacturable pieces.

Keywords: Sculpture design, shape optimization, circle splines, generalized sweeps, saddle surfaces.

1   BACKGROUND AND MOTIVATION

In the mid 1990s I have started to interact and collaborate with Brent Collins, a wood sculptor from Gower, Missouri. Most of Collins' work comprises abstract geometrical shapes that exhibit nicely balanced saddle surfaces. It was originally inspired by natural forms, such as the geometries of soap films suspended in a framework of highly curved boundary wires. In long phone conversations we tried to distill out some generating paradigm that would give us a deeper understanding of these forms and allow us to generate a much wider variety of such shapes in a procedural manner. After a generative concept had been found, the next challenge was to make a judicious selection of parameters that could be varied in an interactive manner and which would allow to cover as wide a domain of possible shapes as possible. The result was *Sculpture Generator I*, a special purpose computer program for the design and visualization Scherk-Collins hole-and-saddle chains warped into toroidal configurations [9,10]. Our first major collaborative sculpture was *Hyperbolic Hexagon II*, shown in Figure 1a.

Fig. 1.   (a) Brent Collins and *Hyperbolic Hexagon II*; (b*) Family of Twelve Trefoils*, models made on a rapid prototyping machine.

This sculpture was designed with *Sculpture Generator I* as a result of extensive consultation with Collins to whom I had sent many pictures of possible design variants. Once we had agreed on the final design, I scaled my virtual model to the desired size of the final sculpture and sliced its geometry at $7/8^{th}$ inch intervals, corresponding to the board thickness from which Collins wanted to assemble the final wood sculpture. I then sent him a roll of 3x3 foot size blue prints that defined the detailed geometry at the proper slicing intervals. Collins cut out these rather convoluted shapes and assembled the 15 board cut-outs properly registered to one another. In a 2-month long carving process he removed the stair casing produced by the boundaries of the cut-out boards and smoothed the surface to perfection. In the meantime I also made many related designs and produced small maquettes on some rapid prototyping (RP) machines. By varying one or two of the dozen parameters available in *Sculpture Generator I*, I could quickly generate whole families of closely related designs (Fig.1b).

2   MINIMAL SURFACES

Many of Collins' sculptures are inspired by the shapes of soap films, and he often chooses surfaces of negative curvature in many of his sculptures. However, the shapes he carves are often quite different from true minimal surfaces with zero mean curvature. Similarly, *Sculpture Generator I* does not really produce minimal surfaces. Its basic geometry module stacks a set of hyperbolas so as to hug a circular opening and constructs a triangulated mesh that combines these profiles into a smooth saddle surface (Fig.2a). This module is then mirrored, rotated, and stacked onto itself to form a Scherk tower (Fig.2b); and this tower can then be twisted and bent into a toroidal ring with a suitable global coordinate transformation – all controlled by about a dozen parameters (Fig.2c). The shapes displayed in Figure 1 use just these simple geometrical operations. However, while they are mostly of negative curvature, they are not truly minimal surfaces.
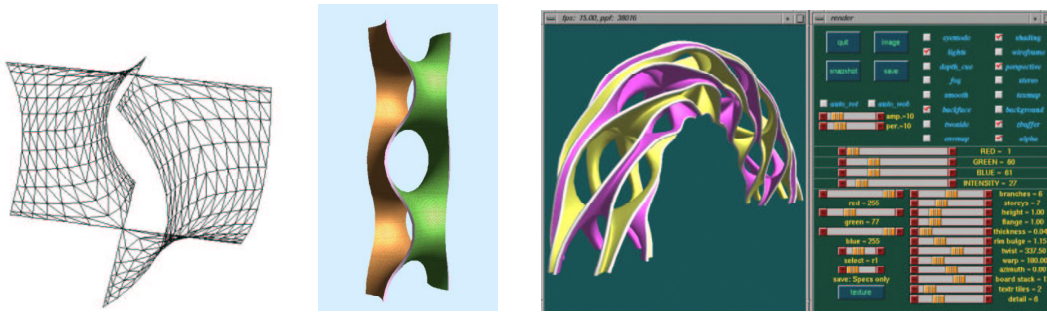

Fig. 2.   *Sculpture Generator I*: (a) the basic module; (b) a composite of these elements into a *Scherk Tower*; (c) user interface menu.

To obtain true minimal surfaces of the same topology, one can subject the triangle meshes to Brakke's *Surface Evolver* [1]. This program will iteratively move all vertices so as to locally minimize surface area, which after an appropriately fine subdivision of the meshes will result in a very good approximation of a minimal surface.

On a couple of occasions I have done this surface optimization and compared the result to the original sculptures. Figure 3a shows a comparison between the *Minimal Trefoil* as it is produced by *Sculpture Generator I*. A true minimal surface as produced by the *Surface Evolver* is shown in Figure 3b. The most noticeable differences are that the optimization reduces the curvature of the flanges in the radial direction to match the curvature in the peripheral direction, and that the central hole closes up to a much smaller size, forming three instances of rather tight saddle configurations. In my opinion, this minimal surface is aesthetically inferior to the original *Trefoil*. Also the tight central saddle configurations cause difficulties for carving such a shape from wood, because it is difficult to ones hands or tolls into these small regions with high curvature. Similarly, in the sculptures to be discussed in the next section, the ribbons want to be curved much more strongly in the lateral direction than in the longitudinal direction – for stability as well as for aesthetic reasons.

However, there are sculptures, where a minimal surface seems to come close to the aesthetic optimum. In the *Volution (Twisted Shell)* series, two examples of which are shown in Figure 3c and 3d, the high genus surfaces were produced with Brakke's *Surface Evolver*, starting from a crude polyhedral base mesh that captured the desired topology and symmetry group.

Fig. 3.   Comparison of (a) the generated *Minimal Trefoil,* and of (b) another version optimized in the *Surface Evolver*. Two types of *Volution* shells optimized in the *Surface Evolver*: (c) inside two cubes, (d) inside an icosahedral path.

3 **COLLINS'** *PAX MUNDI*

During that time Collins also carved *Pax Mundi (World Peace)*, an undulating ribbon following the surface of a sphere (Fig.4a,b). I immediately fell in love with it and also wanted to have a computer program to emulate this shape and to make other sculptures similar to it. However, the scope of *Sculpture Generator I* was much too narrow, and I had to devise a completely new program for this purpose (Fig.4c). The first, and the conceptually most challenging step is to understand a new sculpture in terms of a broader paradigm, so that it can be easily modeled with convenient parameters and that the program then lends itself to the generation of a wide variety of other related sculptures. From the onset it was clear that this sculpture should be modeled as a generalized sweep along a guide curve embedded in the surface of a sphere. It was less clear, how the sweep curve itself should be parameterized.
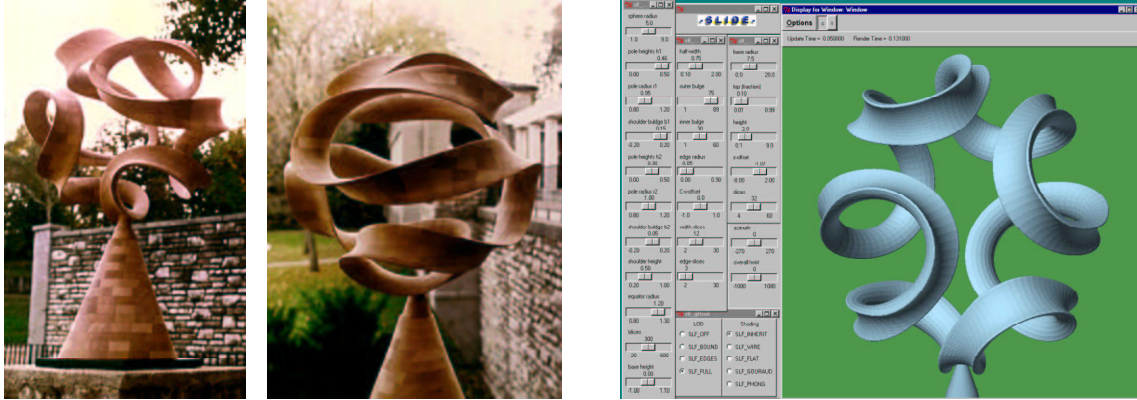

Fig. 4. (a,b) *Pax Mundi* by Brent Collins (199?); and (c) emulation with the *Gabo Sweep Generator* programmed in SLIDE [13].

The breakthrough came when I understood this curve as a generalization of a baseball seam. The undulations also reminded me of the curved edges found in sculptures by Naum Gabo [6]. Thus I formulated the concept of an *n-lobe Gabo curve*, as a meandering path that oscillates around the equator of a sphere, with *n* lobes on either side of it. In its most symmetrical form, it has 2*n* regularly spaced inflection points along the equator and overall exhibits rotational / prismatic symmetry of type D_{nd}. From this perspective, tennis ball or baseball seams are just symmetrical 2-lobe Gabo curves.

To create more artistically interesting sculptures, it is often advantageous to reduce this symmetry and to allow asymmetrically bulging lobes, or to assign different heights to different subgroups of lobes. In this light, *Pax Mundi*, can be understood as an amplitude-modulated 4-lobe Gabo curve (Fig.5a); every second lobe on each hemisphere is of lower amplitude. This leads to the characteristic pattern at the poles of *Pax Mundi*, where two opposing large lobes almost touch, while the two shorter, secondary lobes stay back from this junction.

4   GABO CURVE SWEEP GENERATOR

Because of these considerations, the generator program for this type of ribbon sculpture was broken down into three modules, discussed in the following.
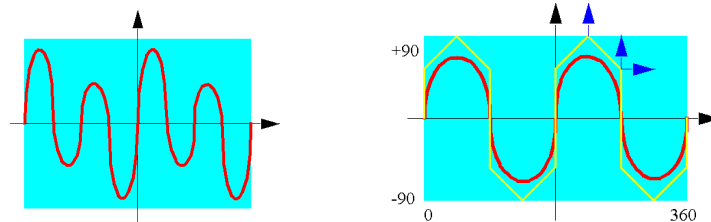

Fig. 5. (a) The curve that was used to emulate *Pax Mundi*; and (b) lobe control in the *Gabo curve generator*.

The Sweep Curve Generator. Originally this was a special purpose program module that allowed the user to specify the number of lobes, their heights, and their widths. A simple B-spline mapped onto the sphere was used. Three numeric parameters per lobe provided sufficient design freedom (small arrows in Fig.5b). Several different lobes can be tied together to the same set of parameters to maintain some regularity or symmetry and to make the design process less laborious. At a

later development stage, an additional parameter was added that allowed to bend the tips of the lobes ever so slightly outwards or inwards towards the center of the sphere. This adjustment was deemed important to allow for the precise visual alignment of the lobes congregating around one of the poles.

The Cross Section Fine-Tuner. This is a plug-in module that generates a cross sectional profile of a particular type. In case of *Pax Mundi* the profile is a crescent with rounded points. A few sliders allow to adjust the size of the crescent, the arc-angle of the dominant inner and outer curves as well as the radius at the tips. All these parameters typically need to be re-adjusted as the sculpture is scaled to a different size or when the fabrication process is changed.

The Sweep / Twist Controller. This module controls the orientation of the cross section along the sweep path. By default, the cross-sectional profile lies in a fixed orientation in the normal plane of the Frenet frame of the sweep curve. This is the case for the *Pax Mundi* sculpture. The concave part of the crescent profile always opens opposite to the local normal vector of the curve. Thus the local curvature of the sweep curve together with the concave part of the profile will produce a continuous elongated saddle of negative Gaussian curvature. This yields an aesthetically pleasing geometry and also provides extra stability for the long ribbon sculpture.

Alternatively, we can select the option to minimize torsion. Now the orientation of the cross section is no longer tied to the Frenet frame, but instead is adjusted in a progressive way so as to minimize the torsion of the ribbon. A small amount of overall twist is introduced to guarantee seamless end-to-end closure of the ribbon. For our particular sweep curve, a nice and consistent shape emerges in which the ribbon is mostly tangent to the sphere (Fig.6a). If we change the azimuth value by 90 degrees, the ribbon takes on a radial orientation (Fig.6b) and it can now be widened considerably without creating self-intersections. The result looks very much like an original Gabo surface [6].
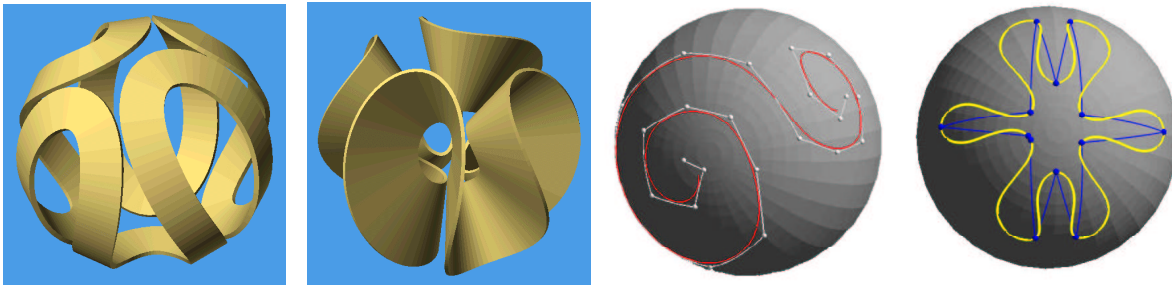

Fig. 6.   Sweep with torsion minimization and two different azimuth values, (a) keeping the ribbon mostly tangential to the base sphere, and (b) normal to it. Curve editors, (c) based on approximating spherical B-splines, and (d) based on interpolating circle splines [].

5   VIAE GLOBI

After having generated a dozen different Gabo sculptures, I wanted to create more convoluted paths on a sphere that could not be described with a simple undulation about the equator. Since I could not think of a general enough procedural description for such curves, I instead set out to define some general-purpose interactive editing tool to create such sweep paths. In particular, I wanted the curves to behave in the same way everywhere on the sphere and not exhibit any special artifacts as the approach the poles; thus standard Euler angles had to be avoided, and ideally the curve description would not refer at all to any particular fixed coordinate system on the sphere.

With the help of Jane Yen, two editors that were free of such singular points have been implemented. The first one generates approximating *spherical B-splines*. It employs the classical de Casteljau algorithm [5] and implements it on the sphere by replacing the construction in Euclidean space based on straight linear interpolation with corresponding great circular arcs. This approach yields very robust and pleasing approximating curves, and since all newly generated subdivision points are always placed on the sphere, the whole final curve is truly embedded in this surface. With this tool, the designer can place an arbitrary sequence of control points on the sphere and then obtains a very smooth **approximating** curve (Fig.6c). The interactive speed of the program allows the user to move the control points until the generated spline curve is aesthetically satisfactory. The second approach generates **interpolating** curves through the given set of points (Fig.6d). It is based on circle splines [11]. For any segment of the control polygon a blend between two circular arcs is generated, and these blends join together with curvature continuity (G^2), or even with parametric C^2-continuity after suitable reparametrization. This approach produces very pleasing "loopy" curves through the given fix-points – exactly the kind that I had been trying to generate with B-splines by carefully adjusting all their control points.

These more free-form curves opened up a whole universe of new sculptural forms. In all my sculptures so far I have maintained one or more C2 rotational symmetry axes. The actual sweep curves used for these sculptures were generated in may different ways. In one instance I just started with a great circle and gradually inserted several hairpin turns and S-curves, resulting in *Stelvio* (Fig.7a), which symbolizes the curvy pathway of an alpine pass road. *Altamont* (Fig.7b) celebrates

American multi-lane freeways; its design started by drawing three parallel lanes on two opposite faces of a cube and then connecting them without any crossings, and finally projecting the result onto a sphere.
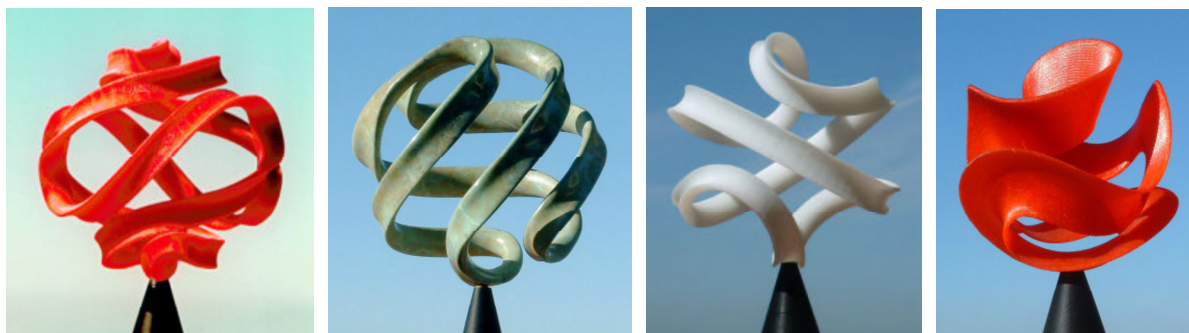

Fig. 7. Examples of *Viae Globi* sculptures: (a) *Stelvio*; (b) *Altamont*; (c) extended 2-lobe Gabo curve; (d) *Aurora_M*

6   FURTHER PARADIGM EXTENSIONS

Whenever I have a parameterized shape generator at my fingertips with which I can generate new possible sculptures in a matter of minutes, I soon start to see boundaries of the domain spanned by that generator as too limiting, and I want to break through those limitations and see what lies beyond. The modular set-up of the Gabo curve sweep generator, together with the extra spherical path-editing tools provide a rather large universe already. Nevertheless, there are some boundaries that can and should be broken. First, our sweep generator allows to control the rotation of the profile as it sweeps along the guide path – either in the form of a continuous twist parameter, or, more locally, by specifying the azimuthal orientation for selected cross sections at some of the control points of the curve; in the latter case, these azimuthal angles get interpolated in exactly the same manner as the x-, y-, and z-coordinates of the control points themselves. The sweep controller also allows the specification of affine deformations of the basic cross sectional profile for each of the curve control points – which again get interpolated by the formula used for the coordinates and the azimuth values. The sweep generator can even morph one profile into another one – provided they both have been described with the same number of data points.

An example of what can be done with these extra features is *Aurora-M* (Fig.7d). It is based on an *enhanced 2-lobe Gabo curve* with two extra loops over the poles (Fig.7c). By adjusting the shape of the cross section and its azimuth, torsion, and twist parameters as it sweeps along the guide curve, a series of shapes emerged that looked like the veils that one might observe in an Aurora Borealis – hence the name. In this particular shape, after torsion minimization, I also introduced a 180º overall twist, which allowed me to keep the ribbon horizontal (tangential) at the base, but vertical (radial) at the top. Furthermore, the cross-section morphs from a rectangle at the top to a crescent at the bottom and back to a rectangle; this enabled me to curve some portion of the ribbon into the shape of a crescent – even though it is a Moebius band.

Another way the extend the *Viae Globi* paradigm is to let the path deviate from the surface of the sphere. Collins has already carved a sculpture like that, called *Music of the Spheres* (Fig.8a) [4]. It was easy to extend my modeling environment to capture that particular sculpture. I just had to use a more general 3D curve editor to produce the original sweep curve. Fortunately the circle spline algorithm seamlessly extends to the unconstrained 3D Euclidean space [], and it can thus readily be used to produce a nice loopy path defined with just a few control points. The path of *Music of the Spheres* stays mostly on the sphere; it just makes three excursions that dive into the interior of the sphere to form three additional loops. With the same generator we can also produce more complicated 3D paths – for instance some that are truly knotted, like the *Chinese Button Knot* (Fig.9b).
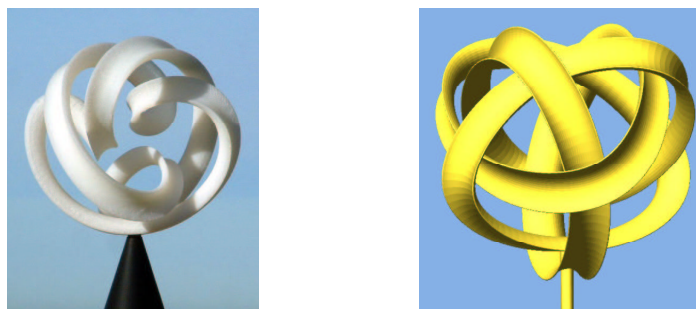

Fig. 8. Sweep paths breaking away from the surface of a sphere: (a) Collins' *Music of the Sphere* [4]; (b) *Chinese Button Knot.*

7 ADAPTATIONS FOR LARGE SCULPTURES

The dream of most artists is to see their work on a grand scale in some public location, where it can be seen by thousands of people. Realizing such sculptures presents a whole set of new problems. In 2006 I had the opportunity to face these new challenges when Collins received a commission to scale up *Pax Mundi* to a bronze sculpture of about 6 feet in diameter as a signature piece for the H&R Block headquarters building in Kansas City.

The first challenge was how to obtain the enlarged master pattern for the scaled-up sculpture. Collins' first idea was to take a 3D scan of his original wood sculpture and enlarge the geometry as needed. But just scaling up proportionally the original *Pax Mundi* sculpture was not appropriate. At the 6-foot scale the relatively wide ribbon of the original wood sculpture would look too clunky. It would also lead to a sculpture that is too heavy (there was a weight limit of 1500 ponds) and too expensive (with the recent price increase for bronze). At that point Collins remembered the existence of my emulation of *Pax Mundi* done with the *Gabo Curve Generator* program, and he asked for my assistance in generating the geometry for the large sculpture. The various parameters in this program allowed us to fine tune the dimensions, shape, and cross section of the ribbon to meet the above weight and price constraints, and to optimize its visual appearance.
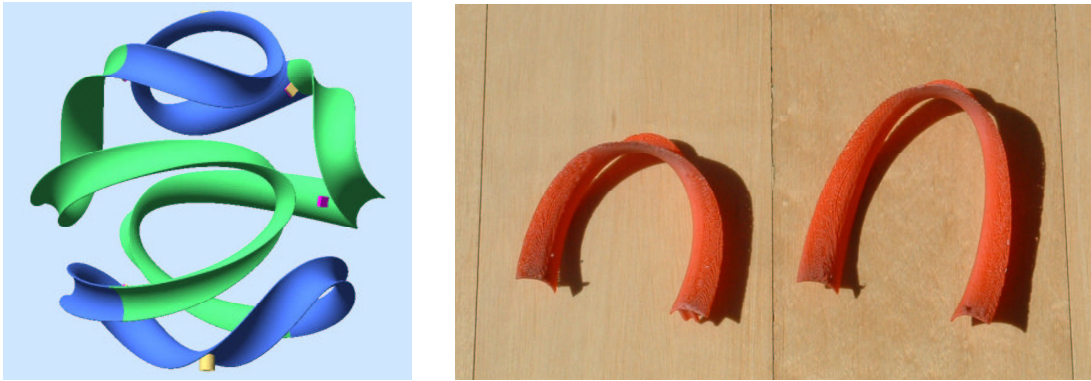

Fig. 9. (a) *Pax Mundi* emulation with four identical segments; (b) master segment split into two U-shaped parts.

It was also clear that such a large sculpture could not be cast as one piece. A small extension to my generator program made it easy to decompose the sculpture into four copies of a single master shape in the form of giant 3-dimensional S-curve (Fig.9a). I had to regroup and relocate the spline control points so as to obtain clean boundary conditions at the joints between the individual segments. Since I anticipated that welding together the individual ribbon sections in a perfectly straight continuous manner would be difficult, I added alignment tabs at the joints (Fig.9a). These allow the pieces to be clamped together in a perfectly aligned position during the welding process (Fig.13a). One of these junctions had an extra large tab which will remain an integral part of the final assembly and will become the support of the sculpture on its pedestal.

Fig. 10. (a) The NC milling machine used to produce master patterns; (b) the smaller master segment obtained by NC milling.

Reducing the number of master patterns that need to be generated is an important economic consideration, since it is a relatively expensive and time consuming part of the process. This single master shape will then be used to make a negative

mold of wood and silicone, which in turn can be used to cast multiple positive copies in wax. These wax replicas of the master shape are then sacrificed in an ordinary investment casting process which creates the four bronze copies needed to assemble the whole sculpture.

But now we faced the problem of how to create this master pattern. No rapid-prototyping machine based on layered manufacturing was available at affordable prices to make this S-shaped master pattern in one piece. Eventually we found an opportunity to create such master patterns at economical rates on a numerically controlled (NC) milling machine, using hard, dense foam plastic (Fig.10a). However, this machine had a limited clearance that could not handle the bulky S-shaped master in one piece. By cutting that shape at the inflection point, we obtained two much flatter U-shaped sub-master patterns that could just fit into the machine (Fig.9b). Extra alignment tabs were added to accommodate the extra junctions and to help in the final assembly.

Reinmuth Bronze Studio was selected to cast the pieces and assemble the final sculpture. It turns out that the milled master patterns, the smaller of which is shown in Figure 10b, were still too large for convenient handling in the bronze foundry. Thus Steve Reinmuth cut the smaller U-shape into two halves, and the larger master pattern into three parts. Before cutting the master patterns apart, Reinmuth made a special jig that would allow him later to recreate those U-shapes perfectly from the individual pieces, even though there were no alignment tabs at the extra junctions introduced by him.

Fig. 11.   (a) One third of the larger master segment used to form a mold; (b) wax copy of this sub-segment with sprues and funnels.

The master patterns were used to create a silicone mold (Fig.11a) in which one could then cast multiple copies in wax. The wax copies were outfitted with the necessary sprues and runners to distribute the liquid bronze quickly to the whole mold cavity, and were also provided with funnels into which the melt could be poured easily (Fig.11b). The enhanced wax segments were then repeatedly dipped into a plaster slurry, until a solid shell had been formed around the whole part (Fig.12a). The plaster shells are then put into an oven preheated to 1800°F, where they sinter into hard forms and where the wax drains out. The fired shells are then filled with liquid bronze of about 2100°F. After the metal has cooled down and hardened, the plaster shell is cracked into small fragments and removed (Fig.12b).

Fig. 12.   (a) Slurry shell around wax copies; (b) removing the plaster shell after the casting process.

The individual pieces, of which there are twenty for *Pax Mundi II*, are cleaned and freed of the extraneous sprues and funnels. Then the assembly process begins (Fig.13a). Some of the smaller pieces are assembled into the U-shapes of the original master patterns. But then the overall assembly process is carried out bottom-up, starting from the point of support, and progressing in a symmetrical manner to guarantee perfect symmetry and balance of the overall sculpture. Since the sculpture is essentially a very long loop that is supported at just one point, the top loop of the ribbon sags by 10 to 20 percent under its own weight. During the assembly process, Reinmuth pre-distorts the shape, so that in its final settled state, the sculpture will be as close to spherical as possible. Figure 13b shows a 5-inch diameter plastic scale model assembled from the four parts depicted in Figure 9a, made on a Fused Deposition Modeling (FDM) machine from Stratasys [14].
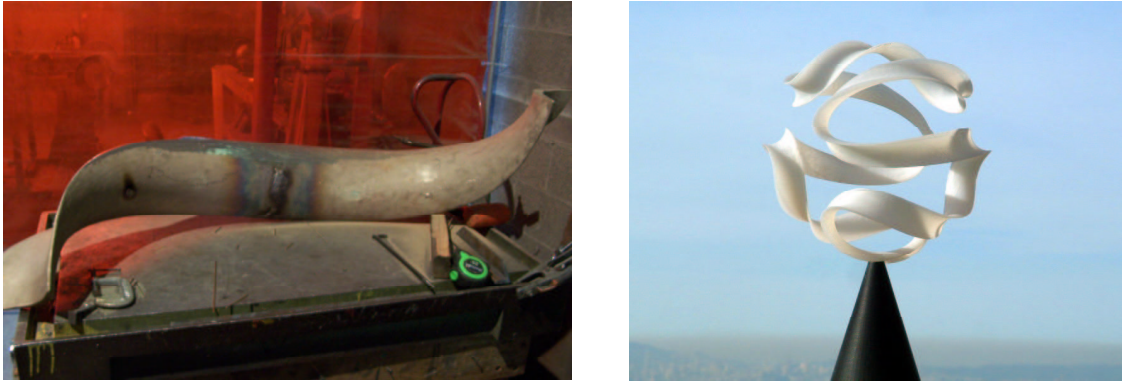

Fig. 13. (a) Two of the cast pieces welded together; (b) model of the whole assembled *Pax Mundi 2*.

At the time of this writing, the about half of the pieces have been welded together. The assembly should be complete in another week. Then the whole ribbon will have to be cleaned and a nice patina applied to the whole surface. Reinmuth is an expert at applying richly textured patinas with a careful mix of chemicals and heat. In the end, the final *Pax Mundi II* will be the result of a truly collaborative effort between the author, Steve Reinmuth, and Brent Collins.

It took us several weeks to figure out how to scale up, redesign, refine, and partition the original model of *Pax Mundi* to make a bronze sculpture of as large a size as possible, but within given weight and budget limits. Some of my pre-existing sculpture generating tools had to be slightly modified to accommodate some of the specific tasks that we encountered in this process, such as partitioning a sculpture into smaller congruent parts. Now, with a much better understanding of the whole process, and with a better infrastructure in place, Brent Collins, Steve Reinmuth and I should be much better equipped to handle similar challenges.

8 CONCLUSIONS

CAD tools have become ubiquitous and pervasive, and are now readily available also to artists. They can play an important role in the entire development cycle of a sculpture, from the initial formation of a concept, through the definition of an optimized form, to the final production of the pieces that make up the actual sculpture. Not all CAD tools are equally suited for all these different phases. In the first phase, one might prefer a computer-aided sketching tool or an interactive editor that helps maintain a chosen symmetry class. In the second phase, good visualization tools are essential, as well as some analytic capabilities that allow to quickly determine the volume (mass) and center of gravity of the emerging sculpture. For the last phase, some robust solid modeling capabilities are required, comprising means to split a sculpture in to smaller parts and to add alignment tabs, or other more sophisticated joint features, to help in the reassembly of the individual pieces. For manufacturing methods where casting or injection molding is involved, tools that simulate or predict crucial aspects of the material flow behavior are needed. One rarely finds all those capabilities in one and the same environment. Several different tools are typically needed – and they don't always exchange information gracefully! Often, communication between the various tools is based on the lowest common denominator, some low-level, redundant boundary representation such as .STL files. Much work still lies ahead for the CAD community to help produce a more integrated environment that will support the creation of artwork or the design and realization of consumer products much more seamlessly – all the way from concept to final realization.

ACKNOWLEDGEMENTS

REFERENCES

[1]   Brakke, K.: The Surface Evolver: *Experimental Mathematics*, Vol. 1, No. 2, pp. 141–165, (1992).
[2]   Collins, B.: Evolving an Aesthetic of Surface Economy in Sculpture. *Leonardo*, Vol. 30, No. 2, 1997, pp. 85-88.
[3]   Collins, B.: Finding an Integral Equation of Design and Mathematics, Proc. Bridges 1988, Fig. 22, p27.
[4]   Collins, B.: Gallery Nine, Ribbon Sculptures: http://www.sckans.edu/~bridges/bcollins/gallery9.html
[5]   de Casteljau, P.: Courbes a poles, *INPI*, 1959.
[6]   Gabo, N.: Translucent Variation on Spheric Theme. 1951, in: *Modern Sculpture*, by H. Read, Thames and Hudson, Inc., New York, Fig. 110.
[7]   Reinmuth Bronze Studio: **http://www.reinmuth.com/**
[8]   Scherk, H. F.: Bemerkungen über die kleinste Fläche innerhalb gegebener Grenzen. *J. Reine Angew. Math. (Crelle's Journal)* 13, pp 185-208 (1835).
[9]   Séquin, C.H.: Virtual Prototyping of Scherk-Collins Saddle Rings. *Leonardo*, Vol. 30, No. 2, 1997, pp. 89-96.
[10] Séquin, C.H.; Meshkin, H.; Downs, L.: Interactive Generation of Scherk-Collins Sculptures. *Symp. on Interactive 3D Graphics*, pp 163-166, Providence RI, April 1997.
[11] Séquin, C.H, Lee, K.; Yen, J.: Fair G2 and C2-Continuous Circle Splines for the Interpolation of Sparse Data Points. *JCAD* Vol 37, No 2, pp 201-211, Feb. 2005.
[12] Séquin, C. H.: CAD Tools for Aesthetic Engineering. *JCAD*, Vol 37, No 7, pp 737-750, June 2005.
[13] SLIDE design environment: http://www.cs.berkeley.edu/~ug/slide/docs/slide/spec/
[14] Stratasys Inc., Fused Deposition Modeling: http://www.stratasys.com/FDM2000.html