Solution to CS375 Homework Assignment 2 (40 points)

Due date: January 28, 2025

1. For each of the following regular expressions find a language (i.e., a set of strings) over A = {a,,b, c} that can be represented/described by that expression. (8 points)

   a. ab* + a*bc   b. a*bbbc*

Either one is okay

Solution:

a. L(ab* + a*bc) = L(ab*) ∪ L(a*bc) = L(a)L(b)* ∪ L(a)*L(b)L(c) ={a}{b}* U {a}*{b}{c}

= {a}{Λ, b, bb, …, b^m, ... } U {Λ, a, aa, …, a^n, … }{bc}

= $\{ab^m \mid m \in N\} \cup \{a^n bc \mid n \in N\} = \{ab^m,\ a^n bc \mid m, n \in N\}$

b. L(a*bbbc*) = L(a*)L(bbb)L(c*) = L(a)*L(b)L(b)L(b)L(c)* = {a}*{b}{b}{b}{c}*

= {a}*{bbb}{c}* = {∧, a, aa, …., a^n, … }{bbb}{∧, c, cc, …., c^m, … }

= { $a^n bbbc^m$ | n, m ϵ N}

Either one is okay

2. Find a regular expression to describe the given language. If it has no regular expression, say so and explain why.

   $\{a, b,\ bac,\ bc,\ b^2ac^2, bc^2,\ \ldots, b^nac^n, bc^m,\ \ldots\}$ (2 points)

Solution:

$\{a, b,\ bac,\ bc,\ b^2ac^2, bc^2,\ \ldots, b^nac^n, bc^m,\ \ldots\}$

= $\{a,\ bac,\ b^2ac^2,\ \ldots, b^nac^n,\ \ldots\} \cup \{b,\ bc,\ bc^2,\ \ldots,\ bc^m,\ \ldots\}$

= $\{\ b^nac^n \mid n \in N\} \cup \{\ bc^m \mid m \in N\ \}$

This language is not a regular language (the second subset is regular, but not the first one), it has no regular expression.

Note that (b*ac* + bc*) is not a regular expression for this language.

3. A regular expression for the language over the alphabet {a, b} with each string containing exactly one ‘ab’ substring is b*a*abb*a*. Use this result to find regular expressions for the following languages

   a. a language over the same alphabet with each string containing two ‘ab’ substrings. (2 points)

   b. a language over the alphabet {a, b, c} with each string containing exactly one ‘abc’ substring. (6 points)

Solution:

a. b*a*abb*a*abb*a*

b. xabcy

where x=y=b*(a + c + cb + ba + bb^+)*b*

4. If a regular expression for the language over the alphabet {a, b} with no string containing the substring aa is (b+ab)*(Λ+a), then what is the regular expression for the language over the alphabet {a, b, c} with no string containing the substring aaa? (4 points)

Solution:

The regular expression is:

(b + c + ab + ac + aab + aac)*(Λ + a + aa)

5. The following proof shows that

   b(a+b)* + bb(a+b)* + bbb(a + b)* = b(a+b)*

Put the reason for each step in the blank on the right-hand side of that step. If an example in the notes can be used for a step, quote that example. (5 points)

b(a+b)* + bb(a+b)* + bbb(a+b)*

= b(a+b)* + (bb+bbb)(a+b)* — · distributes over +

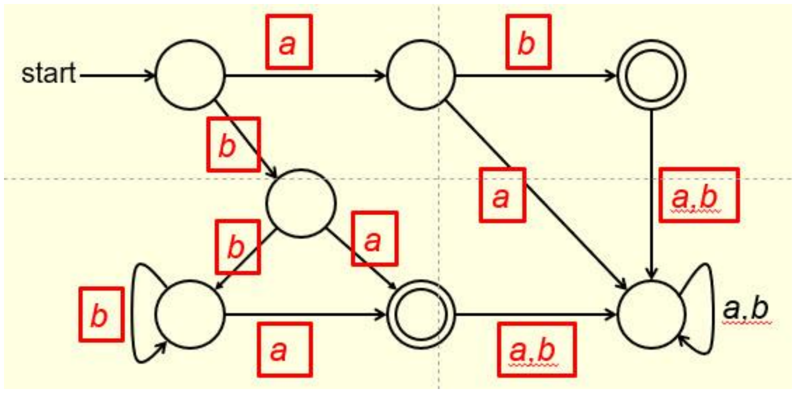
= b(a+b)* + b(b+bb)(a+b)* — · distributes over +

= b(a+b)* + bb(a+b)* — Example in slide 57 of the notes (Formal Languages and Finite Automata I) and the property that (a+b)*= (b+a)*

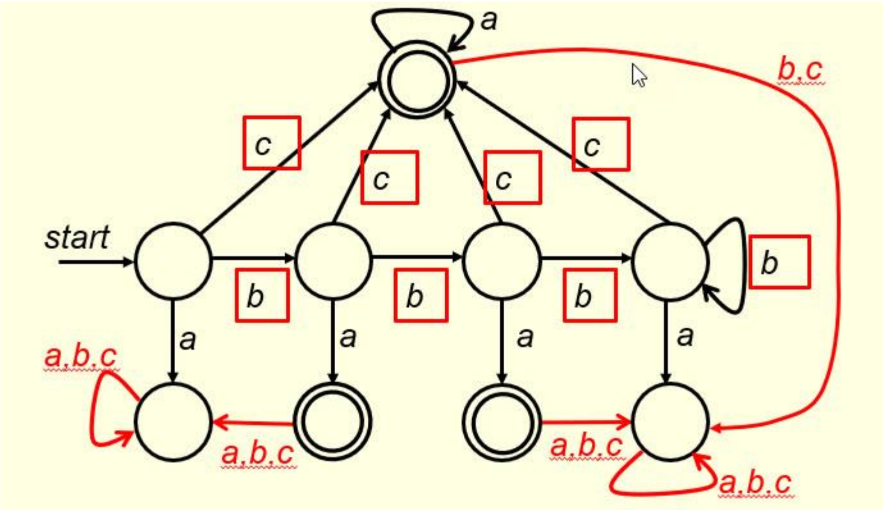
= (b+bb)(a+b)* — · distributes over +

= b(a+b)* — Example in slide 57 of the notes (Formal Languages and Finite Automata I) and the property that (a+b)*= (b+a)*

6. Fill out the blanks in the following figure to make it a DFA that recognizes the expression ab + bb*a. (5 points)



7. Fill out the blanks in the following figure to make it a DFA for the expression b*ca* + bba + ba (4 points)



8. Fill out the blanks in the following figure to make it an NFA for the expression
a* + b*a* + b(a+b)*
If is is possible, simplify the given expression first. (4 points)

Note that a* + b*a* + b(a+b)* = b*a* + b(a+b)*. Hence, we have